ACCURATE EVALUATION OF TRANSIENTS ON DOUBLE CIRCUIT EHV TRANSMISSION LINES

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to the

DEPARTMENT OF ELECTRICAL ENGINEERING

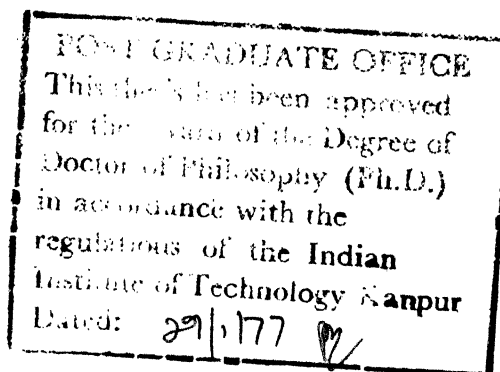**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**
JUNE, 1976

Dedicated to

my parents

CERTIFICATE

Certified that this work, 'Accurate Evaluation of Transients on Double Circuit EHV Transmission Lines' by R. Balasubramanian, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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t = time

ω = angular frequency

a = 'smoothing' factor

$\underline{i}(x,t)$ = current vector at instant t, x units away from the line end

$\underline{v}(x,t)$ = voltage vector at instant t, x units away from the line end

$\underline{V}(x,\omega)$ = modified Fourier transform of $\underline{v}(x,t)$

$\underline{I}(x,\omega)$ = modified Fourier transform of $\underline{i}(x,t)$

$[R]$ = resistance per unit length matrix of the line

$[L]$ = inductance per unit length matrix of the line

$[C]$ = capacitance per unit length matrix of the line

$[Z]$ = transformed series impedance per unit length matrix of the line

$[Y]$ = transformed shunt admittance per unit length matrix of the line

$[P]$ = $[Z][Y]$

$[\gamma]$ = the diagonal propagation constant matrix

$[Q]$ = modal transformation matrix, which makes $[Q]^{-1}[P][Q]$ diagonal

Ω = integration range in the truncated inverse modified Fourier integral

ℓ = length of the line

R_s = source resistance

L_s = source inductance

Any other symbol used is explained at the place where it appears.

SYNOPSIS

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ACCURATE EVALUATION OF TRANSIENTS ON
DOUBLE CIRCUIT EHV TRANSMISSION LINES

An overhead transmission line is subjected to overvoltage transients due to lightning, switching operations or faults. Among these, the overvoltages caused by the lightning is independent of the operating voltage of the line, whereas the overvoltages due to the switching operations or faults depend on the operating voltage of the line. Therefore, oflate as the transmission voltages are increasing, the studies of switching and fault transients have gained more importance. Accurate calculation of these overvoltages permits reduction of safety margins and eventually reduces the cost of insulation of transmission lines and associated equipments.

The various methods of calculating these overvoltage transients, described in the literature may be broadly classified into the lumped parameter methods (including the Transient Network Analyser studies), time domain methods such as Lattice Diagram method, Uram and Miller's method and finite difference method and transform techniques. In this thesis, the modified Fourier transform method, first applied to the power system transient studies by Mullineaux et al and later by Battisson et al and Wedepohl and Mohamad, has been made use of. Briefly

the Fourier transform method of calculating the transient
involves solving the transformed differential equations of
the transmission line for transformed voltage and current at
any point on the line, satisfying the boundary conditions at
the ends of the line by the method of modal analysis and the
transformation of the response to the time domain by perfor-
ming the truncated numerical integration. The resistance
and inductance matrices of a transmission line are in reality
frequency dependent because of the variation of the conductor
resistance with frequency due to the skin effect phenomenon
and the finite conductivity of the earth return path. Hence
an exact study of transients on transmission lines calls for
the inclusion of the effect of frequency dependence of line
parameters. The Fourier transform method, being a frequency
domain method, has the special advantage that the frequency
dependence of line parameters can be directly incorporated
without involving almost any additional computational effort.

The double circuit transmission line is a common
feature of any power system. In the published literature,
except for some TNA studies, the double circuit line transients
have not been studied. Most of the double circuit lines have
their conductors placed at mirror symmetrical positions with
respect to a vertical plane. For a transposed double circuit
line whose conductors are so symmetrically located, a
frequency independent modal transformation is shown to exist
in this thesis. For the untransposed double circuit line of

the above structure, the eigen-value, eigen-vector analysis of two 3x3 matrices are to be performed at each frequency in the integration range to arrive at the modal transformation matrix.

The objective of this thesis is to study the fault and energising transients on a typical double circuit line, using the modified Fourier transform method.

The energisation and re-energisation overvoltages on a transmission line can be contained within the limit as low as 1.5 p.u. by various control methods such as the multi-step resistance closing, controlling the timing of the closure of the individual poles of the circuit breakers etc. When such effective methods of controlling the energisation and re-energisation transient overvoltages are employed, the over-voltages that arise due to fault initiation may become the limiting factor. Ninety percent or more of the faults experienced by a power system are single line to ground faults. In the event of a single line to ground fault, transient overvoltages of the order of 2 p.u. may be experienced by the sound phases. This calls for an accurate calculation of transients due to single line to ground fault initiation. Various authors have studied this problem. Kimbark and Legate have used the TNA to evaluate the fault initiating transient on a single circuit three phase line and have presented a Lattice Diagram approach for a theoretical study of the problem with certain approximations. Later exhaustive analogue computer aided studies of the problem

have been reported. Bonnyubol et al have made use of the
Laplace Transform method and evaluated the fault transient,
employing the 'Residue Theorem' for the inverse Laplace
transformation. Except for a few TNA studies, in all the
above studies, only single circuit lineshave been consi-
dered. Moreover the lines have been assumed to be transposed
and the effect of frequency dependence of line parameters has
not been investigated. In this thesis, the modified Fourier
transform method has been applied to study the transient due
to the initiation of single line to ground fault at any inter-
mediate point on a double circuit transmission line. The need
for the modelling of a double circuit line for fault transient
studies has been established by comparing the fault transients
on single and double circuit lines. A typical 400 KV double
circuit line has been considered, taking into account the
mutual coupling between the two circuits. For the line, fed
on both the ends by inductive sources, the mid span fault
transients have been evaluated, assuming the line to be both
transposed and untransposed. The effect of frequency
dependence of line parameters on the fault transient over-
voltages for both the transposed and untransposed configura-
tions of the line has been investigated. The fault location
is changed to various off-centre points and the fault
transients have been studied. The mid span fault has been
found to give rise to the maximum peak overvoltage.

The energisation of a double circuit line involves
closing of both the circuits. In practice, there is bound

to be a time delay between the closing of the first and second circuits of a double circuit line. This delay is atleast a few seconds if not a few minutes. During this period, the transient due to the energisation of the first circuit would have died down and the first circuit would be in steady state. Thus essentially two transients are involved in the study of the energisation of a double circuit line (viz.) (i) the transient due to the closing of the first circuit, keeping the second circuit unenergised; (ii) the transient due to the closing of the second circuit when the first circuit is already in steady state. The thesis presents the calculation procedure and the results of the above mentioned studies on a double circuit line. A typical 400 KV double circuit line, open at the receiving end has been considered and the voltage waveforms at the open receiving end, subsequent to energisation from an inductive source have been calculated for both transposed and untransposed configurations of the line. The effect of the frequency dependence of line parameters has been incorporated in the studies by using the Carson's formulae for the calculation of the line parameter matrices.

CHAPTER 1

INTRODUCTION

1.1 TRANSIENTS ON TRANSMISSION LINES

The study of electrical transient phenomenon in
power systems has gained great importance, especially with
the increasing voltages of the transmission systems. The
transient overvoltages are developed in a power system
either due to internal or external causes. Lightning over-
voltage is an example of the externally developed over-
voltage and therefore the overvoltage caused by lightning
is independent of the operating voltage of the transmission
system. Internally developed overvoltages are usually
caused by the switching operations (either the opening or
closing of the circuit breakers) and fault initiation or
clearing. The overvoltages due to these depend on the
operating voltage of the system and other system parameters.
In earlier days when the system voltages were lower, the
overvoltages due to lightning were higher than those due to
switching or faults. Nowadays, in high voltage systems,
the overvoltages caused by switching or faults are higher
than those due to lightning. Therefore, the basic insula-
tion level (BIL) of a power system is being determined
more by the behaviour of the system under switching and
fault conditions than by its response to lightning surges.

The cost of insulation of equipment and transmission lines in a high voltage system is very high. When the BIL cannot be determined accurately, the factor of safety has to be rather high. However, accurate determination of the BIL permits the reduction of this factor of safety and thereby the cost of insulation can also be made an optimum. Further, once the magnitude of the overvoltages that can arise in a system is known, methods of controlling them within limits can be developed. An accurate analysis requires an exact modelling of the system components and the physical phenomena involved in the study and similarly an accurate method of solution has to be employed.

When a switching operation or fault takes place, the elements of the power system are subjected to voltages and currents, having a wide frequency range, which may extend from 50 Hz to 100 KHz. The resistance and inductance matrices of a transmission line are, in reality, frequency dependent due to the skin effect phenomenon and the finite conductivity of the earth return path. Many of the EHV transmission lines are untransposed and even if a line is transposed, an ideal transposition is not practically realizable. Hence an exact study of transients on transmission line calls for the inclusion of the effect of frequency dependence of line parameters and handling of the untransposed lines.

There are various methods of calculating these
overvoltage transients on a transmission network, avail-
able in the literature. These may be broadly classified
into lumped parameter method, lattice diagram method,
Uram and Miller's method, Fourier transform method,
Dommel's method and finite difference method. Brief
description of these methods and their salient features
are presented in the following sections.

1.2 LUMPED PARAMETER METHOD

In the lumped parameter method, the transmission
line is represented by a number of lumped T or π sections
and the differential equations of the resulting equivalent
network are solved either with a transient network analyser
or a digital computer.

The favourable features of the method are its
simplicity and its ability to handle untransposed lines
and nonlinear elements directly.

On the other hand, for achieving good accuracy in
the results, a large number of line sections are required
in this method and this correspondingly increases the
storage requirements. The frequency dependence of line
parameters cannot be incorporated in the method. Further,
the computations are to be carried out in stages right
from the initial state. In other words, the response at

any particular instant cannot be directly found out without
going through the calculations in stages from the initial
state.

1.3 LATTICE DIAGRAM METHOD

The Lattice diagram was first introduced by
Bewley [1]. The application of this method to large
scale single and three phase systems has been done by
Barthold and Carter [2], Bickford and Doepal [3] and
others. In this method, lines and cables are specified
by their surge impedances and surge travel times and the
reflected and refracted voltages and currents at junctions
and terminations are calculated by the use of reflection
and refraction coefficients. To reduce the computational
effort, the lumped parameters such as the transformers,
shunt reactors etc. are represented as transmission line
stubs of appropriate reactance values with the terminals
kept open or shorted.

The advantages of this method are that it is
simple, takes less computational effort and can handle
large systems.

On the contrary, in this method, the frequency
dependence of line parameters cannot be incorporated.
Line losses can be taken into account only approximately
by an attenuation factor. Representation of lumped para-
meter elements by stub lines leads to inaccuracy if proper

values of surge impedance and travel time for these stubs
are not chosen. A lot of data such as the reflection
coefficient array, travel time array, the magnitudes of
the forward and backward waves are to be stored. Hence
the memory requirement is quite high.

1.4 URAM AND MILLER'S METHOD

The Uram and Miller's method [4,5] is a time domain
method based on Laplace transform technique. In this
method, by making certain assumptions, an incremental
solution of the transmission line equations is made
possible. Here the voltage and current at each instant
of time and at any point on the transmission line, subse-
quent to a switching operation, can be expressed in terms
of the values of some four functions at that instant
(which can be interpreted as the forward and backward
waves). Among these four functions, two are determined
by the terminal conditions existing at the ends of the
line and the other two are just the attenuated delayed
functions of these two functions. For the case of a line,
which was unenergised before the switching operation, the
delayed functions take zero values initially till the time
equal to the travel time of the line and this facilitates
the starting of an incremental solution of the problem.
For the three phase case, the functions become vectors of
order 3, the components of which correspond to the modal

quantities. The method basically uses Laplace transform technique. The transformed transmission line equation is decoupled into modal components with the help of a constant modal transformation matrix. Such a transformation exists only for the transposed line and hence the applicability of the method is restricted to the transposed lines only. With the other assumption that the resistance offered by the line for the various modes is much less, when compared to the inductive reactance offered by the line for the various modes at all frequencies, the inversion of the response in the Laplace domain to time domain by analytical means is made possible.

The favourable features of the method are that it involves less computational effort and takes less computer time.

But this method cannot take into account the effect of frequency dependence of line parameters directly. The untransposed lines cannot be handled by the method. Computations are to be carried out in stages, starting from the initial state. Nonlinearities cannot be handled directly.

1.5 FOURIER TRANSFORM METHOD

The Fourier transform method has been applied for studying the transients on transmission lines by Day, Mullinaux et al [6-8], Battisson et al [9] and Wedepohl and Mohamed [10]. The method involves solving the Fourier

transformed differential equations of the transmission line for transformed voltage and current at any point on the line, satisfying the boundary conditions at the ends of the line by modal analysis and the transformation of the response to time domain by numerically performing the inverse integral. A modified version of the Fourier transform, which includes a negative exponential term in the basic transform pair relationships makes the integral over the infinite range numerically stable. The infinite integral is truncated at a finite large value for the purpose of performing the numerical integration and the resulting unwanted oscillation (Gibb's oscillation) in the process is avoided by introducing a factor to the integrand. As the method involves calculation of the frequency response in the whole frequency range, the additional calculation of the modal transformation matrix at each frequency to handle the untransposed line case and the calculation of the line parameter values at each frequency to be used in the transients computations are straightforward.

This method does not involve any assumption and represents all the components in a detailed manner. Incorporation of the frequency dependence of line parameters in the method is direct and does not involve any additional computations but for the calculation of just

the parameters at each frequency. Calculation of the
solution just at any desired instant or period can be
done and does not have to be carried out in stages right
from the initial conditions. Handling of lumped parameter
elements is very simple. There is no difficulty in
studying the untransposed line cases by doing the eigen-
value, eigenvector analysis at each frequency to arrive
at the modal transformation matrix at that frequency.

1.6 DOMMEL'S METHOD

The Dommel's solution procedure [11,12] adopts the
method of characteristics [13,14] for the transmission
line and the trapezoidal rule of integration for the
lumped parameter elements. According to the method of
characteristics, for an imaginary observer, moving along
the transmission line in the forward direction with a
velocity equal to that of the travelling waves initiated
on the line, following a disturbance, the expression
(v + z i) remains constant, where v and i are the voltage
and current respectively at any point on the line and z is
the surge impedance of the line. Also for an observer,
moving along the line with the same velocity in the back-
ward direction, the expression (v - z i) remains constant.
Forcing these conditions, the current and voltage at both
the ends of the line can be expressed in terms of the
surge impedance of the line and the past values of the

voltages and currents at these terminals, prior to a
period equal to the travel time of the line. The appli-
cation of the trapezoidal rule of integration for the
lumped parameters facilitates expressing the currents and
voltages at the terminals of these elements at each instant
of time in terms of the values of these currents and
voltages an interval earlier and in terms of the values
of the parameters. Once each component of the network is
replaced by its equivalent circuit described above, the
nodal equations for the whole network at each instant of
time can be written and they can be solved for the unknown
node voltages, knowing the past history of the system over
a time span equal to the travel time for the transmission
line and just the previous step value for lumped parameter
elements. The line losses can be represented by lumped
resistances at the ends and at the middle of the line.
For solving the nodal equations of large scale networks,
the application of the well developed optimally ordered
triangularization and the sparsity techniques reduces the
storage requirement considerably.

This method is well suited for large scale system
problems especially because of the availability of the
well developed sparsity techniques. It can handle non-
linearities. The method is quite general and hence the
development of a general purpose program is possible.

On the other hand, there is no direct way of incor-
porating the frequency dependence of line parameters in this
method. Computations are to be carried out in stages,
starting from the initial state. As the method represents
the lossy nature of the line by lumped parameter elements,
to that extent the modelling is not accurate.

1.7 FINITE DIFFERENCE METHOD

The finite difference method has been applied for
solving the electrical transient problems by Stafford
et al [15] and Raghavan and Sastry [16,17]. In this method
for determining the voltage and current on the transmission
line, which is a function of both distance and time, the
partial differential equations of the line are converted to
difference equations, adopting a central, forward or back-
ward difference scheme. The distance-time space of the
transmission line is divided into a number of lattice
points and the difference equations are solved simultane-
ously step by step to arrive at the voltage and current at
each lattice point, satisfying the boundary conditions at
the ends of the line.

The nonlinearities and untransposed lines can be
handled by this method.

However, proper choice of time step and distance
step is very crucial for ensuring numerical stability. The

method requires large storage. Frequency dependence of line parameters cannot be directly incorporated.

1.8 OBJECTIVES AND OUTLINE OF THE THESIS

The objective of this thesis is mainly to develop and extend the modified Fourier transform method for solving the electrical transients on double circuit transmission lines. A double circuit line is a part and parcel of any power system. In the existing literature, the electrical transients on double circuit lines have been studied mostly using the Transient Network Analyser. This thesis presents the development of a detailed model for a double circuit line and analyses the transients on them due to fault initiation and line energisation by the modified Fourier transform method. The modified Fourier transform method has been rated as one of the more accurate methods for solving these problems as it can be applied to a detailed transmission line model with ease. The distributed nature of all the parameters of the line is preserved in the analysis. The effect of frequency dependence of line parameters can be accounted for easily in the method almost with no additional computational effort. The case of the untransposed line can also be handled by this method.

Also the relevant theory for extending the Uram and Miller's method for studying the electrical transients

on transposed double circuit transmission lines is developed and presented in the thesis. As this method assumes a simpler model for the transmission line, it takes relatively less computer time and gives fairly accurate results. However, it can be used only for a transposed line and without incorporating the effects of frequency dependence of line parameters.

In this chapter a general introduction to the study of electrical transients on transmission systems has been given together with a brief description, and the salient features of the various methods that have been reported in the literature for evaluating these transients.

Chapter 2 describes the theory of the modified Fourier transform method as applied to the study of electrical transients on three phase power systems. The concepts of matrix functions and modal decomposition, proposed by Wedepohl and Mohamed [10], are made use of in the solution procedure. A typical 400 KV single circuit line, open at the receiving end is considered and the transient voltage waveform at the receiving end arising due to its energisation is computed. A comparison of voltage waveforms at the open receiving end for the transposed and untransposed configurations of the line is done.

A mathematical model of a double circuit transmission line, suitable for electrical transient studies is developed

in Chapter 3. It is shown in this chapter that for a transposed double circuit line of mirror symmetric conductor configuration with respect to a vertical plane, there exists a constant modal transformation matrix, which holds good at each frequency of the integration range.

An EHV transmission line is subjected to high overvoltages due to its energisation and re-energisation on trapped charge. Effective ways of restricting these overvoltages to low values, such as multistep resistance switching, controlling the timing of the closure of the individual poles of the circuit breaker etc. have been developed and are implemented in practice to limit these overvoltages to less than 2 p.u. When such methods are employed for controlling switching overvoltages, the over-voltages that arise due to fault initiation and fault clearing may dominate the scene over the switching over-voltages. Because of this, the study of fault transients on transmission lines is of importance. In Chapter 4, the application of the modified Fourier transform method for evaluating the overvoltage transients, arising on the sound phases of a transmission line due to a single phase to ground fault at any intermediate point on the line is described. The mathematical model, developed in Chapter 3 for a double circuit line is used to study the transient overvoltages on the sound phases due to a single phase to

ground fault on one circuit of a typical 400 KV double
circuit line. A comparison of fault overvoltages for the
transposed and untransposed configurations of the line is
made. The effect of frequency dependence of line parameters
on the fault initiating transient is studied for both the
transposed and untransposed double circuit lines. The
fault transient is evaluated for various intermediate
fault locations.

The modified Fourier transform method is applied
to compute the energisation transients on a double circuit
line in Chapter 5. The model, developed in Chapter 3 for
the double circuit line is used in these studies. The
energisation of the first circuit with the second circuit
remaining unenergised as well as the energisation of the
second circuit when the first circuit is operating in
steady state are considered. In these studies, it is
assumed that all the three poles of the circuit breaker
of each circuit get closed simultaneously. This simpli-
fies the analysis and helps in getting an idea of the
magnitudes of the overvoltages involved. A 400 KV line
example is considered and the energisation transient at
its open receiving end is computed for both the transposed
and untransposed configurations of the line.

The extension of the Uram and Miller's method of
solving the electrical transients for a double circuit

line is done in Chapter 6. The existence of a constant modal transformation matrix, independent of frequency, for a transposed double circuit line, as shown in Chapter 3, has made the application of this method for this problem possible. This being a time domain method, it takes much less computer time but can handle only transposed lines and the frequency dependence of line parameters cannot be directly incorporated. The method is applied to the same 400 KV double circuit line example as of Chapter 5. The results, obtained for the example for both the first and second circuit energisation are presented.

In Chapter 7, the effect of the nonsimultaneous closing of the circuit breaker poles on the energisation transients is investigated. The theory of the modified Fourier transform method is extended for analysing the energisation transients on double circuit lines, incorporating the sequential closing of breaker poles. The same 400 KV double circuit line example as of Chapters 5 and 6 is considered and the energising transients are evaluated for both the transposed and untransposed configurations of the line. The effect of frequency dependence of line parameters on the transients is investigated.

In the concluding chapter 8, the results of the various chapters of the thesis are reviewed, conclusions are drawn and **prob**lems for further research are outlined.

CHAPTER 2

MODIFIED FOURIER TRANSFORM METHOD

2.1 INTRODUCTION

The theory of the modified Fourier transform method
and its application for solving the electrical transient
problems [9,10] have been described in this chapter. This
method has been adopted in this work with a view to repre-
sent the transmission line and other components in a
detailed manner so that the transients can be evaluated
more accurately. It is easy to incorporate the effect
of frequency dependence of line parameters in this method
of transients evaluation. The method takes all the para-
meters of the transmission line to be distributed and can
handle untransposed lines. To illustrate the method, an
example of a single circuit line has been considered and
the results of the energisation study are presented.

2.2 THEORY OF THE METHOD

The solution of the transients on multiconductor
transmission lines is complicated by the fact that the
partial differential equations describing the behaviour
of the system are 4-dimensional (3-dimensions in space
and the time dimension). By making the assumption that
only plane-wave propagation takes place, it is possible
to separate the two space variables mutually at right

angles to the direction of propagation and to express them
in terms of equivalent impedances, as described by Carson
[18]. This leaves space variable in the direction of
propagation and the time variable alone in the analysis.
With this assumption, the voltage and current at any point
on a transmission line are determined by the partial
differential equations

$$-\frac{\partial v(x,t)}{\partial x} = R\ i(x,t) + L\ \frac{\partial i(x,t)}{\partial t} \qquad (2.1)$$

$$-\frac{\partial i(x,t)}{\partial x} = G\ v(x,t) + C\ \frac{\partial v(x,t)}{\partial t} \qquad (2.2)$$

where $v(x,t)$, $i(x,t)$ are the voltage and current respec-
tively at the point, x units away from the end at time 't'

    R is the resistance/unit length of the line

    L is the inductance/unit length of the line

    G is the conductance to ground/unit length of the line

and   C is the capacitance to ground/unit length of the line.

For a transmission line, usually G is negligibly
small and hence (2.2) takes the form

$$-\frac{\partial i(x,t)}{\partial x} = C\ \frac{\partial v(x,t)}{\partial t} \qquad (2.3)$$

When equations (2.1) and (2.3) are Fourier trans-
formed, they become full differential equations in trans-
formed voltage and current.

The Fourier transform pair is defined by the
expressions

$$F(\omega) = \int_0^\infty f(t)\ e^{-j\omega t}\ dt \qquad\qquad (2.4)$$

$$f(t) = \frac{1}{2\pi}\ \int_{-\infty}^\infty F(\omega)\ e^{j\omega t}\ d\omega \qquad\qquad (2.5)$$

Equations (2.1) and (2.3), when Fourier transformed take the form

$$-\frac{dV(x,\omega)}{dx} = [R + j\omega L]\ I(x,\omega) \qquad\qquad (2.6)$$

$$-\frac{dI(x,\omega)}{dx} = j\omega C\ V(x,\omega) \qquad\qquad (2.7)$$

Differentiating equation (2.6) with respect to x and substituting for $\frac{dI(x,\omega)}{dx}$ from equation (2.7), we get

$$\frac{d^2 V(x,\omega)}{dx^2} = [R + j\omega L]\ j\omega C\ V(x,\omega) \qquad\qquad (2.8)$$

Solution of the above equation can be written as

$$V(x,\omega) = \text{Cosh } \gamma x \cdot A + \text{Sinh } \gamma x \cdot B \qquad\qquad (2.9)$$

where $\quad \gamma = ([R + j\omega L]j\omega C)^{\frac{1}{2}}$

and A and B are arbitrary constants, which are determined by the terminal conditions, existing at the ends of the line.

From equation (2.6), the expression for transformed current can be obtained as

$$I(x,\omega) = -[\frac{j\omega C}{(R+j\omega L)}]^{\frac{1}{2}}[\text{Cosh } \gamma x \cdot B + \text{Sinh } \gamma x \cdot A] \qquad (2.10)$$

Once the boundary conditions are known in the frequency domain, the arbitrary constants A and B can be evaluated and the transformed voltage and current at any point on the line can be found out with the help of equations(2.9) and (2.10). As this response in frequency domain is a complicated expression involving hyperbolic functions, it can be transformed back to time domain only by numerically evaluating the inverse integral of the form (2.5). The above mentioned conventional Fourier transform technique is not suitable for the evaluation of this integral numerically, as the integrand in this form is not quite stable numerically. A negative exponential term, when introduced to the integrand makes it numerically stable and hence a modified version of the transform is preferred for the calculations.

The modified Fourier transform pair is given by the expressions

$$V(x,\omega) = \int_{0}^{\infty} v(x,t)\ e^{-(a+j\omega)t}\ dt \qquad (2.11)$$

$$v(x,t) = \frac{1}{2\pi}\ \int_{-\infty}^{\infty} V(x,\omega)\ e^{(a+j\omega)t}\ d\omega \qquad (2.12)$$

where a is a positive constant, sufficiently large to ensure that the above integrals converge.

When the modified Fourier transform is applied instead of the conventional Fourier transform,

equations (2.6) and (2.7) get modified to the form

$$-\frac{dV(x,\omega)}{dx} = [R + (a + j\omega)L] \, I(x,\omega) \qquad (2.13)$$

$$-\frac{dI(x,\omega)}{dx} = (a + j\omega)C \, V(x,\omega) \qquad (2.14)$$

Putting these equations together, we get

$$\frac{d^2V(x,\omega)}{dx^2} = Z \, Y \, V(x,\omega) \qquad (2.15)$$

where $\qquad Z = R + (a + j\omega)L$

and $\qquad Y = (a + j\omega)C.$

The general solution for transformed voltage and current at any point on the line can now be written as

$$V(x,\omega) = \text{Cosh } \gamma x \cdot A + \text{Sinh } \gamma x \cdot B \qquad (2.16)$$

$$I(x,\omega) = -(Y/Z)^{\frac{1}{2}} \, [\text{Cosh } \gamma x \cdot B + \text{Sinh } \gamma x \cdot A] \qquad (2.17)$$

where now $\gamma = \{ [R + (a + j\omega)L](a + j\omega)C \}^{\frac{1}{2}}$.

The boundary conditions, when substituted in equations (2.16) and (2.17) determine the constants A and B. The final step is to evaluate the integral (2.12) to arrive at the response in time domain.

For evaluating the above integral numerically, the infinite range is, in practice, truncated at a large finite value, say Ω. To effect economy in computation, a half range form of the Fourier integral can be used and in that

case the integral to be evaluated takes the form

$$v(x,t) = \frac{2e^{at}}{\pi} \int_{0}^{\Omega} \text{Real Part}[V(x,\omega)]\text{Cos}\omega t \, d\omega$$

The truncation of the infinite range of the integral at a finite value has been found to give rise to unwanted oscillations, known as Gibb's oscillation in the solution. To eliminate these oscillations, a weighting function known as sigma factor is introduced to the integrand. With the introduction of this factor, the integral reads as

$$v(x,t) = \frac{2e^{at}}{\pi} \int_{0}^{\Omega} \text{Real Part}[V(x,\omega)]\text{Cos}\omega t \, \frac{\text{Sin}(\frac{\pi\omega}{\Omega})}{(\frac{\pi\omega}{\Omega})} d\omega \quad (2.18)$$

A set of suitable parameters ($\Omega, \Delta\omega$ and a) to achieve accuracy and economy in computation, while evaluating numerically the above integral has been reported [6-8].

Accuracy in representation of the rise time of the response is determined, in main, by the choice of Ω. For a good representation, it is significant to choose Ω so that the calculated rise time is less than 10 percent of the travel time of the transmission line.

i.e. $T < \tau/10$

It has been found, in practice, that the inaccuracy in the rise time is principally in the range $\Omega T > 4$. So the above criterion implies choosing Ω so that

$$\Omega T > 4$$

i.e. $\Omega > 40/\tau$

It has been confirmed by experience that the angular frequency interval $\Delta\omega = \Omega/500$ is a good choice.

While the parameter 'a' can be used to 'smoothen' the term $V(a,\omega)$ in the integrand, the 'Cosωt' term oscillates with a period in ω of $2\pi/t$. For taking into account this oscillation, introduced to the integrand accurately, it is desirable to choose $\Delta\omega$, so that at least 4 points occur in each quarter cycle

i.e. $16\Delta\omega < 2\pi/t$

In other words, the period over which the response is calculated should be restricted for a particular value of $\Delta\omega$ chosen so that

period of calculation $< 0.4/\Delta\omega$

2.3 THREE PHASE SYSTEMS – MODAL ANALYSIS

The theory involved in applying the modified Fourier transform method for solving the transients on single phase systems has been presented in the previous section. When this is extended to a 3 phase system, the voltages and currents become vectors and the line parameters become matrices.

Now the matrix differential equation, which has to be solved to arrive at the transformed voltage vector at any point on the line takes the form

$$\frac{d^2 \underline{V}(x,\omega)}{dx^2} = [P]\,\underline{V}(x,\omega) \qquad (2.19)$$

where $\qquad [P] = [Z][Y]$

$\qquad\qquad [Z] = [R] + (a + j\omega)[L]$

and $\qquad [Y] = (a + j\omega)[C]$

For solving the coupled matrix differential equation (2.19), the concept of matrix function is to be made use of. By the concept of matrix function, if a mathematical operation is to be performed on a square matrix, the resulting matrix after the operation can be obtained by doing the operation individually on the eigenvalues (modal values) of the matrix and by pre-multiplying and post-multiplying the diagonal matrix, having these operated eigenvalues as diagonal elements, by the eigenvector (or modal transformation) matrix and its inverse respectively.

Symbolically,

$$f([A]) = [Q] \, f([\lambda]) \, [Q]^{-1}$$

Using this property, the solution of equation (2.19) can be written as

$$V(x,\omega) = [\text{Cosh } \psi x] \, \underline{A} + [\text{Sinh } \psi x] \, \underline{B} \qquad (2.20)$$

where $\qquad [\psi] = [Q][\gamma][Q]^{-1}$

$$[\text{Cosh } \psi x] = [Q][\text{Cosh } \gamma x][Q]^{-1}$$

$$[\text{Sinh } \psi x] = [Q][\text{Sinh } \gamma x][Q]^{-1}$$

$[\gamma]$ is the diagonal matrix, whose diagonal elements are the square roots of the eigenvalues of matrix $[P] = [Z][Y]$

[Q] is the modal transformation matrix, which makes

$[Q]^{-1}[P][Q]$ diagonal

and $\underline{A}$ and $\underline{B}$ are arbitrary constant vectors, which can be evaluated by the terminal conditions of the line.

The general current vector solution can now be determined as

$$\underline{I}(x,\omega) = -[Z]^{-1}[\psi]\{[\text{Cosh } \psi x] \ \underline{B} + [\text{Sinh } \psi x] \ \underline{A} \} \qquad (2.21)$$

When the boundary conditions are forced into equations (2.20) and (2.21), the constant vectors $\underline{A}$ and $\underline{B}$ are determined and hence the transformed voltage and current vectors at any point on the line is completely determined. Then the final step of evaluating the inverse integral of each of the phase components of the transformed response separately for the 3 phase case yields the phase voltages and currents in time domain.

In the above analysis, if the transmission line is transposed, the computation of the eigenvalues γ's and eigenvector [Q] becomes unnecessary, as for this case a constant modal transformation, which holds good at each frequency in the integration range exists. For the transposed line, the line parameter matrices and hence the matrix [P] have a special structure, in which all the diagonal elements are equal and all the off-diagonal elements are equal. Though the values of these diagonal and off-diagonal elements vary with frequency, still at

each frequency this special structure of equality of diagonal and off-diagonal elements is preserved. The eigenvalues of such a matrix [P] are the one corresponding to the ground or zero sequence mode, given by (diagonal element + 2 x off-diagonal element) and the other two (repeated ones) corresponding to the aerial modes given by (diagonal element – off-diagonal element). An eigenvector matrix for the matrix [P] of such a structure [4] is given by

$$[Q] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix} \qquad (2.22)$$

When the line is untransposed, the matrix [P] does not possess this special structure. Hence for this case the eigenvalue, eigenvector analysis has to be performed at each frequency to arrive at the matrices [Q] and [γ], which hold good only at that frequency.

2.4 MATHEMATICAL MODEL FOR ENERGISATION STUDY

In this section, the development of an algorithm for computing the transient following the energisation of a single circuit, three phase line [10], applying the theory of the previous two sections is described. The algorithm assumes simultaneous closing of the circuit breaker poles during energisation.

Supposing the transformed voltages at the sending
and receiving ends of a transmission line are known, forcing
of these conditions in the general voltage vector solution
expression (2.20) yields

$$\mathbf{A} = \underline{V}_S \qquad\qquad (2.23)$$

$$[\mathrm{Cosh}\psi\ell]\underline{V}_S + [\mathrm{Sinh}\,\psi\ell]\underline{B} = \underline{V}_R$$

where ℓ is the length of the line,

i.e. $\underline{B} = [\mathrm{Cosech}\ \psi\ell]\underline{V}_R - [\mathrm{Coth}\,\psi\ell]\underline{V}_S \qquad (2.24)$

From (2.21), (2.23) and (2.24), the transforms of currents
at the terminals of the line can be expressed in terms of
the terminal transformed voltages as follows:

$$
\begin{bmatrix} \underline{I}_S \\ \underline{I}_R \end{bmatrix}
=
\begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{12}] & [Y_{11}] \end{bmatrix}
\begin{bmatrix} \underline{V}_S \\ \underline{V}_R \end{bmatrix}
\qquad (2.25)
$$

where $\qquad [Y_{11}] = [Z]^{-1}[\psi][\mathrm{Coth}\,\psi\ell]$

and $\qquad [Y_{12}] = -[Z]^{-1}[\psi][\mathrm{Cosech}\,\psi\ell]$

In the above equation, the matrix relating the
terminal voltages to the terminal currents can be inter-
preted as the short circuit admittance parameter matrix of
the line, viewed as a two port network.

Supposing a shunt element is added at one of the
terminals of the transmission line, then the short circuit
admittance matrix of the resulting overall set up can be

obtained by just adding to the self admittance term,
corresponding to the particular terminal of the transmis-
sion line, the admittance value of the added shunt element.

For simulating the energisation of the line from a
constant voltage source having a source impedance, conversion
of the voltage source with the series impedance to its
equivalent current source with the appropriate shunt admit-
tance facilitates the application of the modified short
circuit admittance parameter matrix (obtained as described
above) in the analysis.

Figure 2 .1.1 and 2.1.2 give the system set up
with voltage source and with the equivalent current source
respectively.

Referring to Figure 2.1.2 when the shunt elements
corresponding to the source admittances are added at the
sending end terminals, the short circuit admittance para-
meter of the system takes the form

$$\begin{bmatrix} ([Y_{11}] + [Y_s]) & [Y_{12}] \\ [Y_{12}] & [Y_{11}] \end{bmatrix} \qquad (2.26)$$

where

$$[Y_s] = \begin{bmatrix} 1/[R_s+(a+j\omega)L_s] & 0 & 0 \\ 0 & 1/[R_s+(a+j\omega)L_s] & 0 \\ 0 & 0 & 1/[R_s+(a+j\omega)L_s] \end{bmatrix}$$

$$(2.27)$$
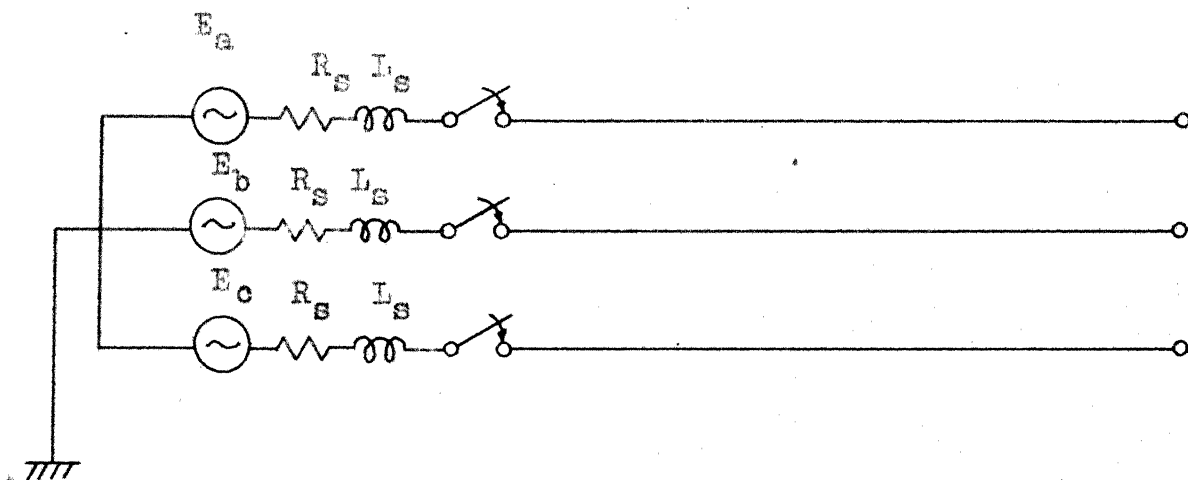
FIGURE 2.1.1: SYSTEM DIAGRAM WITH SOURCE REPRESENTATION

$$I_a = \frac{E_a}{[R_s + (a+j\omega)L_s]}$$

$$I_b = \frac{E_b}{[R_s + (a+j\omega)L_s]}$$
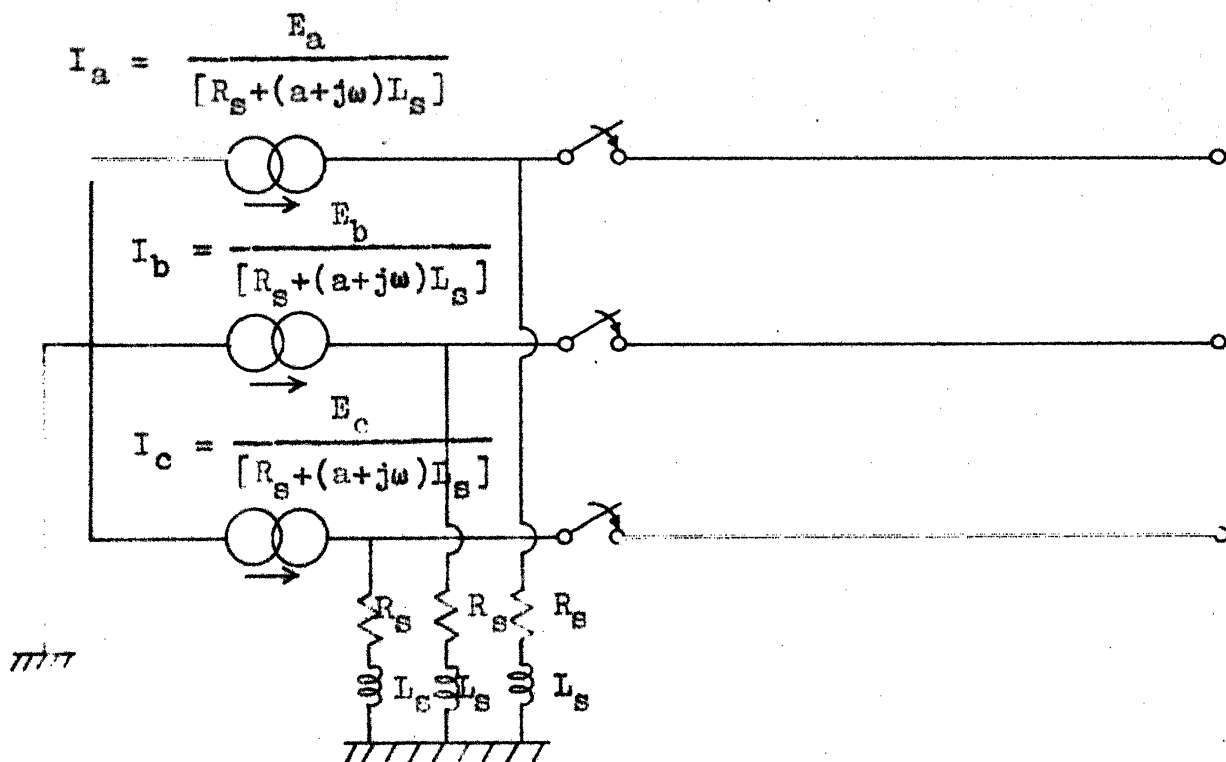
$$I_c = \frac{E_c}{[R_s + (a+j\omega)L_s]}$$

FIGURE 2.1.2: SYSTEM DIAGRAM WITH EQUIVALENT
CURRENT SOURCE.

Now the desired response is calculated for the injected current vector at the sending end

$$\underline{I}_s = \begin{bmatrix} E_A/[R_s+(a+j\omega)L_s] \\ E_B/[R_s+(a+j\omega)L_s] \\ E_C/[R_s+(a+j\omega)L_s] \end{bmatrix} \qquad (2.28)$$

where E_A, E_B and E_C are the modified Fourier transform of the source voltages of the three phases, which are analytically known.

Considering the case of a line, open at the receiving end, equation (2.25), when the boundary conditions are substituted take the form

$$\begin{bmatrix} \underline{I}_s \\ \underline{0} \end{bmatrix} = \begin{bmatrix} ([Y_{11}] + [Y_s]) & [Y_{12}] \\ [Y_{12}] & [Y_{11}] \end{bmatrix} \begin{bmatrix} \underline{V}_s \\ \underline{V}_R \end{bmatrix} \qquad (2.29)$$

From the above equation, expressions for $\underline{V}_s$ and $\underline{V}_R$ can be obtained in terms of $\underline{I}_s$ as follows:

$$\underline{V}_s = \{[Y_{11}]+[Y_s]-[Y_{12}][Y_{11}]^{-1}[Y_{12}]\}^{-1}\,\underline{I}_s \qquad (2.30)$$

$$\underline{V}_R = \{[Y_{12}]-([Y_{11}]+[Y_s])[Y_{12}]^{-1}[Y_{11}]\}^{-1}\,\underline{I}_s \qquad (2.31)$$

2.5 NUMERICAL EXAMPLE

The above algorithm is applied to study the transient following the energisation of a typical 400KV single circuit, three phase line, open at the receiving end and the results of the study are presented in this

TIME, MS

TRANSPOSED
UNTRANSPOSED
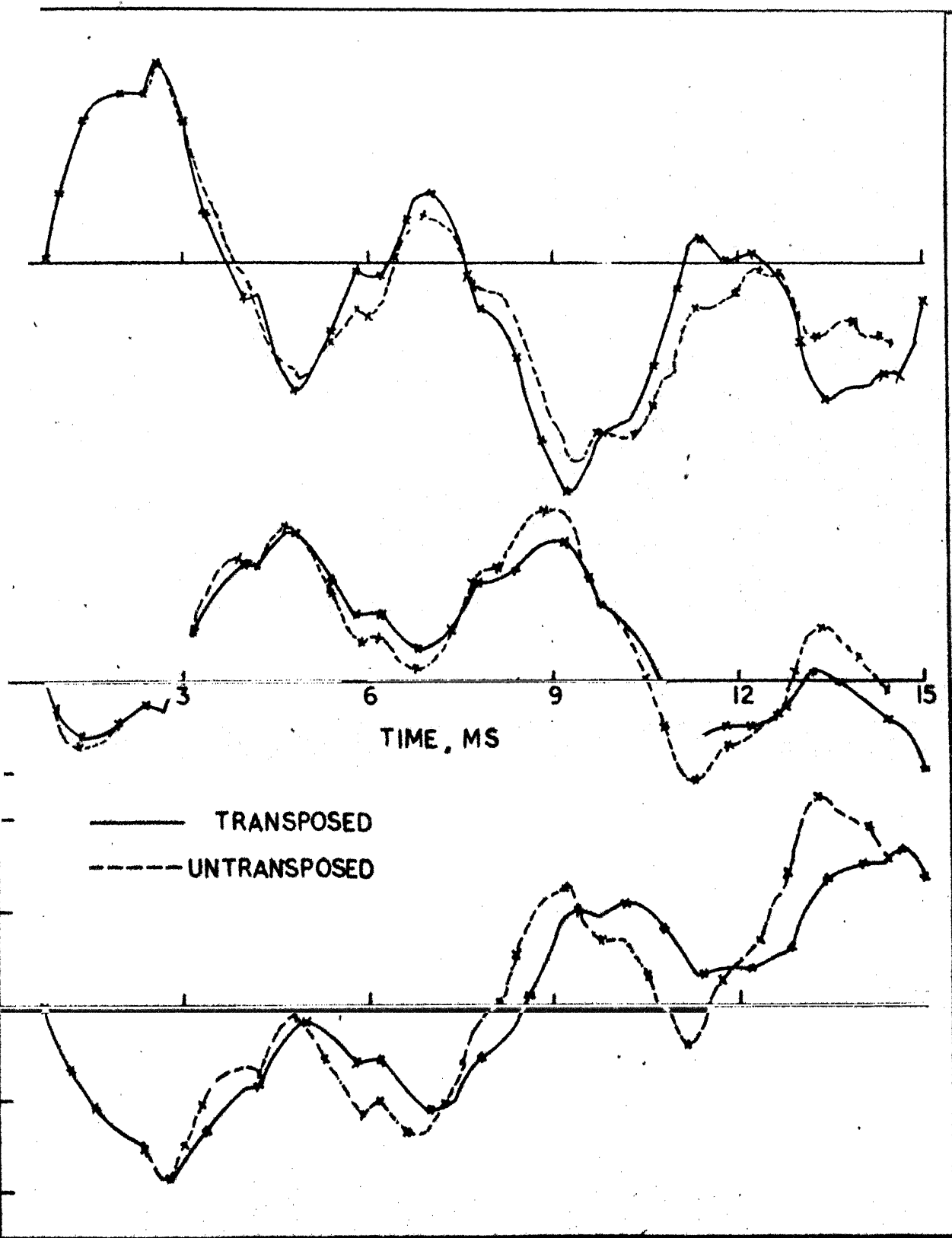
FIGURE 2.1: SWITCHING TRANSIENT VOLTAGE WAVEFORMS AT RECEIVING END.

section. The source has been assumed to be purely inductive. The effect of the presence of earth wire is included by making the necessary correction to the phase parameter matrices of the line. The line parameters and other system data are given in Appendix A.

The phase voltage waveforms at the open receiving end, following the energisation for the transposed and untransposed configurations of the line are compared in Figure 2.2. All the three poles of the circuit breaker are assumed to close simultaneously at the instant when the source voltage of phase-A is at its peak value. A maximum peak of -2.45 p.u. is observed to occur on phase- A at 9.2 m.secs. for the case of the transposed line, whereas the maximum peak for the case of the untransposed line is 2.22 p.u. on phase- C at 13.26 m.secs.

2.6 CONCLUSIONS

The theory of modified Fourier transform method for studying the electrical transients on transmission lines has been presented in this chapter. A typical 400KV, single circuit, three phase line example has been consi- dered and the transients following its energisation from an inductive source has been evaluated for both the cases of transposed and untransposed configurations of the line. The extension of the theory for evaluating the energisation and fault initiating transients on double circuit transmis- sion lines is described in the following chapters.

CHAPTER 3

MODELLING OF A DOUBLE CIRCUIT LINE
FOR TRANSIENT STUDIES

3.1 INTRODUCTION

In this chapter, a model of a double circuit transmission line, suitable for transient studies is developed. This model is used in the subsequent chapters for analysing transients due to energisation and fault initiation. The double circuit line is a part and parcel of any power system. So far, in the literature, except for a few studies using Transient Network Analyser (TNA) [19], the double circuit line transients have not been reported.

The study of transients on a transmission line involves solving of the polyphase wave equations. The first step in the solution procedure is to decompose the equation to modal components. The transformation matrix for this purpose is to be obtained by performing the eigenvalue, eigenvector analysis of the line propagation coefficient matrix.

In any practical power system, most of the double circuit lines have got a mirror symmetric conductor configuration with respect to a vertical plane. This gives most economical tower configuration. For such a special conductor configuration, a saving in the computation of the eigenvalues and eigenvector can be

achieved [20]. In the case of an ideally transposed
double circuit line of mirror symmetric conductor confi-
guration, a constant modal transformation matrix, which
holds good at all frequencies can be shown to exist.

3.2 I MODAL TRANSFORMATION FOR A DOUBLE CIRCUIT TRANSMISSION LINE

The line parameter matrices of a double circuit
transmission line, whose conductor configuration is mirror
symmetric with respect to a vertical plane are symmetrical.
The matrix [P] of the line, which is given by [Z][Y] will,
therefore also be symmetrical. Further the matrix [P]
or the line parameter matrices, when partitioned, take
the special form

$$[M] = \begin{bmatrix} [M_a] & [M_b] \\ [M_b] & [M_a] \end{bmatrix}$$

In the case of [P], this may be written as

$$[P] = \begin{bmatrix} [P_a] & [P_b] \\ [P_b] & [P_a] \end{bmatrix} \qquad (3.1)$$

whereas [P_a] and [P_b] are matrices of order 3. These
submatrices [P_a] and [P_b] are symmetrical. However,
their diagonal elements are unequal. Similarly, their
off-diagonal elements are also unequal.

The eigenvalues of the matrix $[P]$ are obtained by solving the polynomial equation

$$\det([P] - \lambda_i [U2]) = 0 \qquad (3.2)$$

for the six values λ_i; $i = 1, 2, \ldots, 6$, where $[U2]$ is unit matrix of order 6.

Since the determinant of a matrix product is equal to the product of the determinants of the component matrices, taken in any order, if we choose a particular regular matrix

$$[K] = \begin{bmatrix} [U] & -[U] \\ [U] & [U] \end{bmatrix} \qquad (3.3)$$

where $[U]$ is unit matrix of order 3, the equation for finding the eigenvalues can be written as

$$\det\{[K]^{-1}([P] - \lambda_i[U2]) [K]\} = 0 \qquad (3.4)$$

It can be verified that the matrix product

$$[K]^{-1}([P] - \lambda_i[U2])[K] = \begin{bmatrix} ([P_a] + [P_b] - \lambda_{si}[U]) & [0] \\ [0] & ([P_a] - [P_b] - \lambda_{di}[U]) \end{bmatrix}$$

where

$$[0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ,$$

λ_{si}; $i = 1, 2, 3$ are the eigenvalues of the matrix $([P_a] + [P_b])$

and λ_{di}; $i = 1,2,3$ are the eigenvalues of the matrix $([P_a]-[P_b])$.

Therefore, equation (3.4) becomes

$$\det \begin{bmatrix} ([P_a]+[P_b]-\lambda_{si}[U]) & [0] \\ [0] & ([P_a]-[P_b]-\lambda_{di}[U]) \end{bmatrix} = 0$$

i.e. $\det ([P_a]+[P_b]-\lambda_{si}[U]) \; \det ([P_a]-[P_b]-\lambda_{di}[U]) = 0$

$$(3.5)$$

Hence the eigenvalues of the matrix $[P]$ can be obtained by solving the equations

$$\det ([P_a]+[P_b]-\lambda_{si}[U]) = 0 \quad \text{for } \lambda_{si} \; ; \; i=1,2,3$$

and $\quad \det ([P_a]-[P_b]-\lambda_{di}[U]) = 0 \quad \text{for } \lambda_{di} \; ; \; i=1,2,3$

separately.

The eigenvector matrix $[Q]$ of the matrix $[P]$ is to be obtained by solving the matrix equation

$$[P][Q] = [\lambda][Q] \qquad\qquad (3.6)$$

where $[\lambda]$ is the diagonal eigenvalue matrix of matrix $[P]$.

Equation (3.6) can be manipulated to the form

$$[K]^{-1}[P][K][K]^{-1}[Q] = [K]^{-1}[\lambda][K][K]^{-1}[Q]$$

where the matrices $[K]$ and $[K]^{-1}$ are the same as described earlier.

We have seen that

$$[K]^{-1}([P]-[\lambda])[K] = \begin{bmatrix} ([P_a]+[P_b]-[\lambda_s]) & [0] \\ [0] & ([P_a]-[P_b]-[\lambda_d]) \end{bmatrix}$$

where $[\lambda_s]$ is the diagonal eigenvalue matrix of the matrix $([P_a]+[P_b])$ and $[\lambda_d]$ is the diagonal eigenvalue matrix of the matrix $([P_a]-[P_b])$.

Let $[K]^{-1}[Q]$ be denoted by $[Q']$. Then

$$\begin{bmatrix} ([P_a]+[P_b]-[\lambda_s]) & [0] \\ [0] & ([P_a]-[P_b]-[\lambda_d]) \end{bmatrix} \begin{bmatrix} [Q'_{11}] & [Q'_{12}] \\ [Q'_{21}] & [Q'_{22}] \end{bmatrix}$$

$$= \begin{bmatrix} [0] & [0] \\ [0] & [0] \end{bmatrix} \qquad (3.7)$$

where $[Q'_{11}]$, $[Q'_{12}]$, $[Q'_{21}]$ and $[Q'_{22}]$ are submatrices, obtained after partitioning of matrix $[Q']$ for conformable multiplication in the above equation.

From equation (3.7), it is evident that

if $\det([P_a]+[P_b]-[\lambda_s]) = 0$

and $\det([P_a]-[P_b]-[\lambda_d]) \neq 0$,

then $[Q'_{21}] = [0]$

and $[Q'_{11}]$ is obtained by solving the matrix equation

$$([P_a]+[P_b]-[\lambda_s])[Q'_{11}] = [0].$$

Similarly if $\det([P_a]-[P_b]-[\lambda_d]) = 0$

and $\det([P_a]+[P_b]-[\lambda_s]) \neq 0$,

then $[Q'_{12}] = [0]$

and $[Q'_{22}]$ is obtained by solving the matrix equation

$$([P_a]-[P_b]-[\lambda_d])[Q'_{22}] = [0].$$

Thus $[Q']$ takes the form

$$[Q'] = \begin{bmatrix} [Q'_{11}] & [0] \\ [0] & [Q'_{22}] \end{bmatrix}$$

The above form of the matrix $[Q']$ is very useful. It contains only the self term $[Q'_{11}]$ and $[Q'_{22}]$. The mutual terms $[Q'_{12}]$ and $[Q'_{21}]$ are zero. This fact is of particular importance in the present calculations. Therefore,

$$[Q] = [K][Q'] = \begin{bmatrix} [U] & -[U] \\ [U] & [U] \end{bmatrix} \begin{bmatrix} [Q'_{11}] & [0] \\ [0] & [Q'_{22}] \end{bmatrix}$$

$$= \begin{bmatrix} [Q'_{11}] & -[Q'_{22}] \\ [Q'_{11}] & [Q'_{22}] \end{bmatrix} \qquad (3.8)$$

The above matrix $[Q]$ again contains only the self term $[Q'_{11}]$ and $[Q'_{22}]$ and the mutual terms $[Q'_{12}]$ and $[Q'_{21}]$ are not present. Therefore, the problem of solving the transient on a double circuit line can be reduced to the

solution of two single circuit lines, decoupled from each other. Thus, all operations and manipulations can be carried out independently on each circuit as if the other does not exist. Only when the composite parameters are required, the independent values are combined by means of matrix [K].

3.2.1 Transposed Line:

For an ideally transposed line, the submatrix $[P_a]$ takes the particular form, in which all the diagonal elements are equal and all the off-diagonal elements are also equal. In addition, all the elements of the submatrix $[P_b]$ are also equal. Hence the matrices $([P_a]+[P_b])$ and $([P_a]-[P_b])$ will also have the structure, in which all the diagonal elements are equal and all the off-diagonal elements are equal. The diagonal and off-diagonal elements of these matrices $([P_a]+[P_b])$ and $([P_a]-[P_b])$ (which are dependent upon the line parameter matrices) vary in magnitude with the frequency. However, the diagonal elements of each of these matrices separately will be equal. Similarly, the off-diagonal elements of each of these matrices will also be separately equal.

e.g.
$$\begin{bmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{bmatrix}$$

For a 3x3 matrix of such a structure, there exists a frequency independent eigenvector matrix (vide Section 2.3)

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

Hence for the case of an ideally transposed double circuit line, the matrices $[Q'_{11}]$ and $[Q'_{22}]$ in equation (3.8) take the frequency independent form

$$[Q'_{11}] = [Q'_{22}] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

Thus a transformation matrix, which decouples the [P] matrix for the case of the transposed double circuit line at each frequency, inspite of the variation of [P] matrix with frequency, is given by

$$\begin{bmatrix} 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

3.2.2 Untransposed Line:

For the case of an untransposed double circuit line, the eigenvalue, eigenvector analyses [21] for the matrices $([P_a]+[P_b])$ and $([P_a]-[P_b])$ are to be performed at each frequency to obtain the modal transformation matrix [Q].

3.3 CONCLUSIONS

This chapter describes the development of a modal transformation matrix for a double circuit transmission line of mirror symmetric conductor configuration with respect to a vertical plane. If the double circuit line is ideally transposed, it has been shown that a constant modal transformation matrix, independent of frequency, exists. This reduces the computational effort; needed for the calculation of transients in the case of a transposed double circuit line. This model has been made use of for analysing energising and fault transients on double circuit lines in the subsequent chapters.

CHAPTER 4

FAULT INITIATING TRANSIENT ON A
DOUBLE CIRCUIT TRANSMISSION LINE

4.1 INTRODUCTION

Switching overvoltages on an EHV transmission line arise due to energisation and re-energisation of the line. In practice, it is desirable to contain these overvoltages to a value as low as possible in order to reduce the cost of insulation. The various control methods such as the multistep resistance switching, controlling the timing of the closure of the individual poles of the circuit breaker etc. are effective in restricting these overvoltages to a value as low as 1.5 p.u. When such effective methods of controlling overvoltages are employed, the overvoltages that arise due to fault initiation may become a limiting factor. Ninety percent or more of the faults, experienced by a power system are single line to ground faults. In the event of a single line to ground fault, transient over-voltages of the order of 2 p.u. may be experienced by the sound phases. This calls for an accurate evaluation of transients on a transmission line due to single line to ground fault initiation.

The transient due to fault initiation on single circuit transmission line has been studied by various authors. Kimbark and Legate [22] have used the TNA to

evaluate the fault transient and have presented a lattice diagram approach for a theoretical study of the problem with certain approximations. Later exhaustive analogue computer studies of the problem have been reported [23,24]. Boonyubol et al [25] have made use of the Laplace transform method and have calculated the fault transient, employing the residue theorem for the inverse Laplace transformation. In all the above studies, the lines were assumed to be ideally transposed and the line parameters, computed at a single frequency have been assumed to remain constant in the whole frequency range.

The double circuit transmission line is a common feature of any power system. In this chapter, the application of modified Fourier transform method for studying the transients due to fault initiation on a double circuit line is described. A typical double circuit line has been considered, taking into account the mutual coupling between the two circuits. A single line to ground fault is applied on phase 'a' at any point along its length and the resulting overvoltages on the sound phases are calculated. Both the cases of an ideally transposed line and a completely untransposed line are considered. For the case of an ideally transposed double circuit line, a frequency independent modal transformation exists, as shown in Chapter 3 and hence the calculation of the transient for this case involves much

less computation. The frequency dependence of line para-
meters is included, using Carson's formulae [18] (Appendix B).

4.2 SIMULATION OF FAULT

The principle of superposition has been applied for
simulating fault on the transmission line. The system
considered is linear and therefore superposition can be
applied to the problem. Initially before the fault occurs,
the line has been assumed to be under steady state.

The steps involved in the simulation of fault can
be explained with the help of Figure 4.1 as follows:

(i) First assuming the fault not to be present, the
voltage at the fault point and the required response at any
point of interest on the line are determined as functions
of time. As the line is in steady state, these functions
are pure sinusoids at power frequency.

(ii) As the second step, the voltage at the fault point
on the faulted phase, calculated in step (i) is injected as
cancellation voltage with reversed polarity, across the
faulted phase and ground, at the same point. The source
voltages at the ends of the line are short circuited. Now
the transient response of the line to this injected voltage
alone is calculated as a function of time, starting from
the instant of fault initiation (taken as zero time).

(iii) The superposition of these two responses gives
the total response as a function of time, subsequent to
the initiation of the fault.
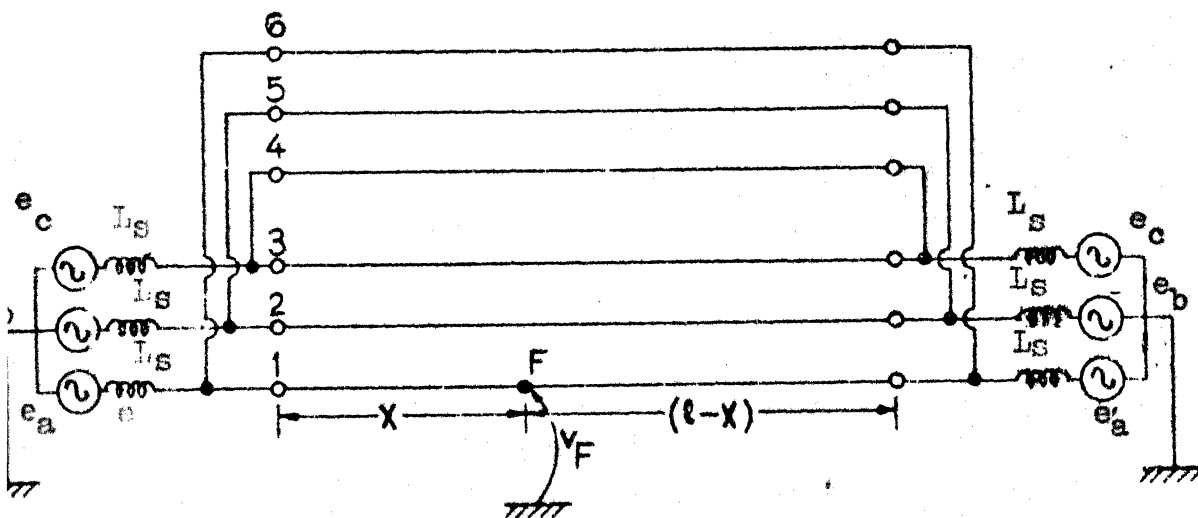
FIGURE 4.1.1
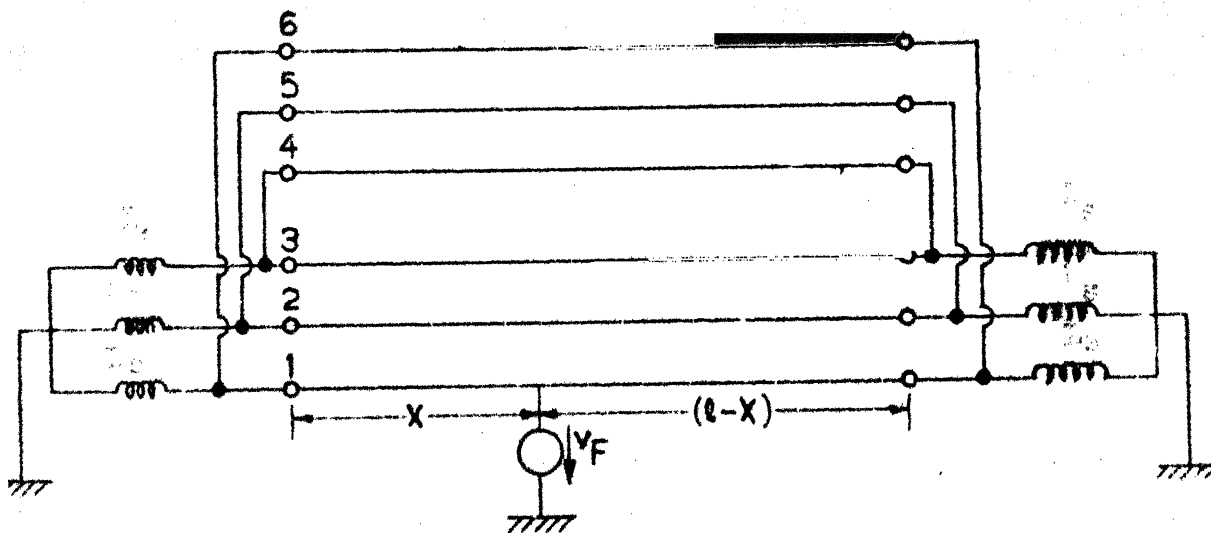
FIGURE 4.1.2

PRINCIPLE OF SUPERPOSITION USED FOR FAULT SIMULATION

The steady state voltages to be evaluated in step (i) are nearly 1 p.u. The calculation procedure for the evaluation of the transient response to the injected voltage of step (ii) is described in the next section.

4.3 EVALUATION OF TRANSIENT RESPONSE FOR THE INJECTED VOLTAGE

The general solution of the transformed transmission line equations for the transformed phase voltage and current vectors at any point, x units away from the source end is given by

$$\underline{V}(x,\omega) = [\text{Cosh}\,\psi x]\,\underline{A} + [\text{Sinh}\,\psi x]\,\underline{B} \qquad (4.1)$$

$$\underline{I}(x,\omega) = -[Z]^{-1}[\psi]\{[\text{Cosh}\,\psi x]\,\underline{B} + [\text{Sinh}\,\psi x]\,\underline{A}\} \qquad (4.2)$$

where $[\psi] = [Q][\gamma][Q]^{-1}$

$[\text{Cosh}\,\psi x] = [Q][\text{Cosh}\,\gamma x][Q]^{-1}$

$[\text{Sinh}\,\psi x] = [Q][\text{Sinh}\,\gamma x][Q]^{-1}$

$[\gamma]$ = the diagonal matrix, whose diagonal elements are the square roots of the eigenvalues of the matrix $[P] = [Z][Y]$

$[Q]$ = the eigenvector matrix (modal transformation matrix), which makes $[Q]^{-1}[P][Q]$ diagonal

and $\underline{A}$ and $\underline{B}$ are arbitrary constant vectors, which have to be evaluated by forcing the terminal conditions of the line.

Referring to Figure 4.1, let the fault be located at a distance X units from the left end. The line section to

the left of the fault point F is considered first. The
terminal conditions at the left end of the line are as
follows:

(i) As the two circuits are paralleled at this end,
the transformed voltages of the connected phases of each
of the circuits at this end, obtained by substituting
$x = X$ in the general solution expression (4.1) should be
equal.

(ii) The transformed phase currents through the
terminating source impedances at this end are the sum of
the transformed currents fed by the corresponding phases
of both the circuits, calculated by substituting $x = X$ in
the general expression for current (4.2). Further, the
transformed voltages and currents at this end of the line
should satisfy the transformed voltage-current relationship
of the terminating impedances.

These conditions can be put down mathematically in
the following form:

Let $[CH]$ $= [Cosh\ \psi X]$

$[SH]$ $= [Sinh\ \psi X]$

$[SC]$ $= -[Z]^{-1}\ [\psi][Cosh\ \psi X]$

$[SS]$ $= -[Z]^{-1}\ [\psi][Sinh\ \psi X]$

Then denoting the elements of the above matrices
by lower case letters in double subscript notation

$$\begin{bmatrix} ch_{11} & ch_{12} & \cdots & ch_{16} \\ ch_{21} & ch_{22} & \cdots & ch_{26} \\ ch_{31} & ch_{32} & \cdots & ch_{36} \end{bmatrix} \underline{A} + \begin{bmatrix} sh_{11} & sh_{12} & \cdots & sh_{16} \\ sh_{21} & sh_{22} & \cdots & sh_{26} \\ sh_{31} & sh_{32} & \cdots & sh_{36} \end{bmatrix} \underline{B}$$

$$= \begin{bmatrix} ch_{61} & ch_{62} & \cdots & ch_{66} \\ ch_{51} & ch_{52} & \cdots & ch_{56} \\ ch_{41} & ch_{42} & \cdots & ch_{46} \end{bmatrix} \underline{A} + \begin{bmatrix} sh_{61} & sh_{62} & \cdots & sh_{66} \\ sh_{51} & sh_{52} & \cdots & sh_{56} \\ sh_{41} & sh_{42} & \cdots & sh_{46} \end{bmatrix} \underline{B}$$

$$(4.3)$$

$$\left\{ \begin{bmatrix} (ss_{11}+ss_{61}) & \cdots & (ss_{16}+ss_{66}) \\ (ss_{21}+ss_{51}) & \cdots & (ss_{26}+ss_{56}) \\ (ss_{31}+ss_{41}) & \cdots & (ss_{36}+ss_{46}) \end{bmatrix} \underline{A} \right.$$

$$\left. + \begin{bmatrix} (sc_{11}+sc_{61}) & \cdots & (sc_{16}+sc_{66}) \\ (sc_{21}+sc_{51}) & \cdots & (sc_{26}+sc_{56}) \\ (sc_{31}+sc_{41}) & \cdots & (sc_{36}+sc_{46}) \end{bmatrix} \underline{B} \right\} (a+j\omega) L_s$$

$$= \begin{bmatrix} ch_{11} & ch_{12} & \cdots & ch_{16} \\ ch_{21} & ch_{22} & \cdots & ch_{26} \\ ch_{31} & ch_{32} & \cdots & ch_{36} \end{bmatrix} \underline{A} + \begin{bmatrix} sh_{11} & sh_{12} & \cdots & sh_{16} \\ sh_{21} & sh_{22} & \cdots & sh_{26} \\ sh_{31} & sh_{32} & \cdots & sh_{36} \end{bmatrix} \underline{B}$$

$$(4.4)$$

where L_s is the source inductance.

From (4.3) and (4.4), $\underline{B}$ and $\underline{A}$ can be related by the expression,

$$\underline{B} = [T_X] \underline{A} \tag{4.5}$$

where $[T_X] = [T_{X1}]^{-1}[T_{X2}]$

$$[T_{X1}]=\begin{vmatrix}
[sh_{11}-(a+j\omega)L_s(sc_{11}+sc_{61})] & \cdots & [sh_{16}-(a+j\omega)L_s(sc_{16}+sc_{66})] \\
[sh_{21}-(a+j\omega)L_s(sc_{21}+sc_{51})] & \cdots & [sh_{26}-(a+j\omega)L_s(sc_{26}+sc_{56})] \\
[sh_{31}-(a+j\omega)L_s(sc_{31}+sc_{41})] & \cdots & [sh_{36}-(a+j\omega)L_s(sc_{36}+sc_{46})] \\
(sh_{61}-sh_{11}) & \cdots & (sh_{66}-sh_{16}) \\
(sh_{51}-sh_{21}) & \cdots & (sh_{56}-sh_{26}) \\
(sh_{41}-sh_{31}) & \cdots & (sh_{46}-sh_{36})
\end{vmatrix}$$

$$[T_{X2}]=\begin{vmatrix}
[(a+j\omega)L_s(ss_{11}+ss_{61})-ch_{11}] & \cdots & [(a+j\omega)L_s(ss_{16}+ss_{66})-ch_{16}] \\
[(a+j\omega)L_s(ss_{21}+ss_{51})-ch_{21}] & \cdots & [(a+j\omega)L_s(ss_{26}+ss_{56})-ch_{26}] \\
[(a+j\omega)L_s(ss_{31}+ss_{41})-ch_{31}] & \cdots & [(a+j\omega)L_s(ss_{36}+ss_{46})-ch_{36}] \\
(ch_{11}-ch_{61}) & \cdots & (ch_{16}-ch_{66}) \\
(ch_{21}-ch_{51}) & \cdots & (ch_{26}-ch_{56}) \\
(ch_{31}-ch_{41}) & \cdots & (ch_{36}-ch_{46})
\end{vmatrix}$$

From the general solution expressions (4.1) and (4.2) and equation (4.5), the input admittance of the left section of the line in frequency domain, when viewed from the fault point is

$$[Y_X] = -[Z]^{-1}[\psi][T_X] \tag{4.6}$$

By adopting a similar procedure, the input admittance of the right section of the line in the frequency domain, viewed from the fault point can be proved to be

$$[Y_{\ell-X}] = -[\psi][\ ]^{-1}[\psi][T_{\ell-X}] \tag{4.7}$$

where $[T_{\ell-X}]$ is obtained by substituting $(\ell - X)$ in the place of X in the expression for $[T_X]$.

The total input admittance of the line as viewed from the fault point $[Y] = [Y_X] + [Y_{\ell-X}]$.

Now at the fault point, the transformed voltage on the faulted phase, WV_F (the modified Fourier transform of the injected voltage source) is known and the injected currents on the sound phases are zero.

Forcing these conditions, we get

$$[\ ]\begin{bmatrix} V_F \\ V_b \\ V_c \\ V_{c'} \\ V_{b'} \\ V_{a'} \end{bmatrix} = \begin{bmatrix} I_a \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4-8}$$

From the above equation, the transformed voltages on the sound phases at the fault point are given in terms of the known quantity V_F by the expression

$$
\begin{bmatrix} V_b \\ V_c \\ V_{c'} \\ V_{b'} \\ V_{a'} \end{bmatrix} = -V_F \begin{bmatrix} y_{22} & y_{23} & \cdots & y_{26} \\ y_{32} & y_{33} & \cdots & y_{36} \\ y_{42} & y_{43} & \cdots & y_{46} \\ y_{52} & y_{53} & \cdots & y_{56} \\ y_{62} & y_{63} & \cdots & y_{66} \end{bmatrix}^{-1} \begin{bmatrix} y_{21} \\ y_{31} \\ y_{41} \\ y_{51} \\ y_{61} \end{bmatrix} \qquad (4.9)
$$

The transient response, given above in the frequency domain is then transformed back to the time domain by performing the numerical integration.

4.4 VERIFICATION OF THE COMPUTER PROGRAM

To check the computer program, developed for the fault study, the single circuit line examples, studied by Boonyubol et al [25] is solved. The data of the example is given in Appendix C.

The voltage waveforms at the fault point on the sound phases are plotted in Figure 4.2. These results agree well with those of Boonyubol et al.

4.5 SYSTEM USED FOR THE STUDY

A typical 400 KV double circuit transmission line of configuration [10], shown in Figure 4.3 is next analysed for the evaluation of fault initiating transient. The line is fed on both the ends by similar sources. In the example considered, both the source sides are represented purely by lumped impedances. The values of these source impedances are
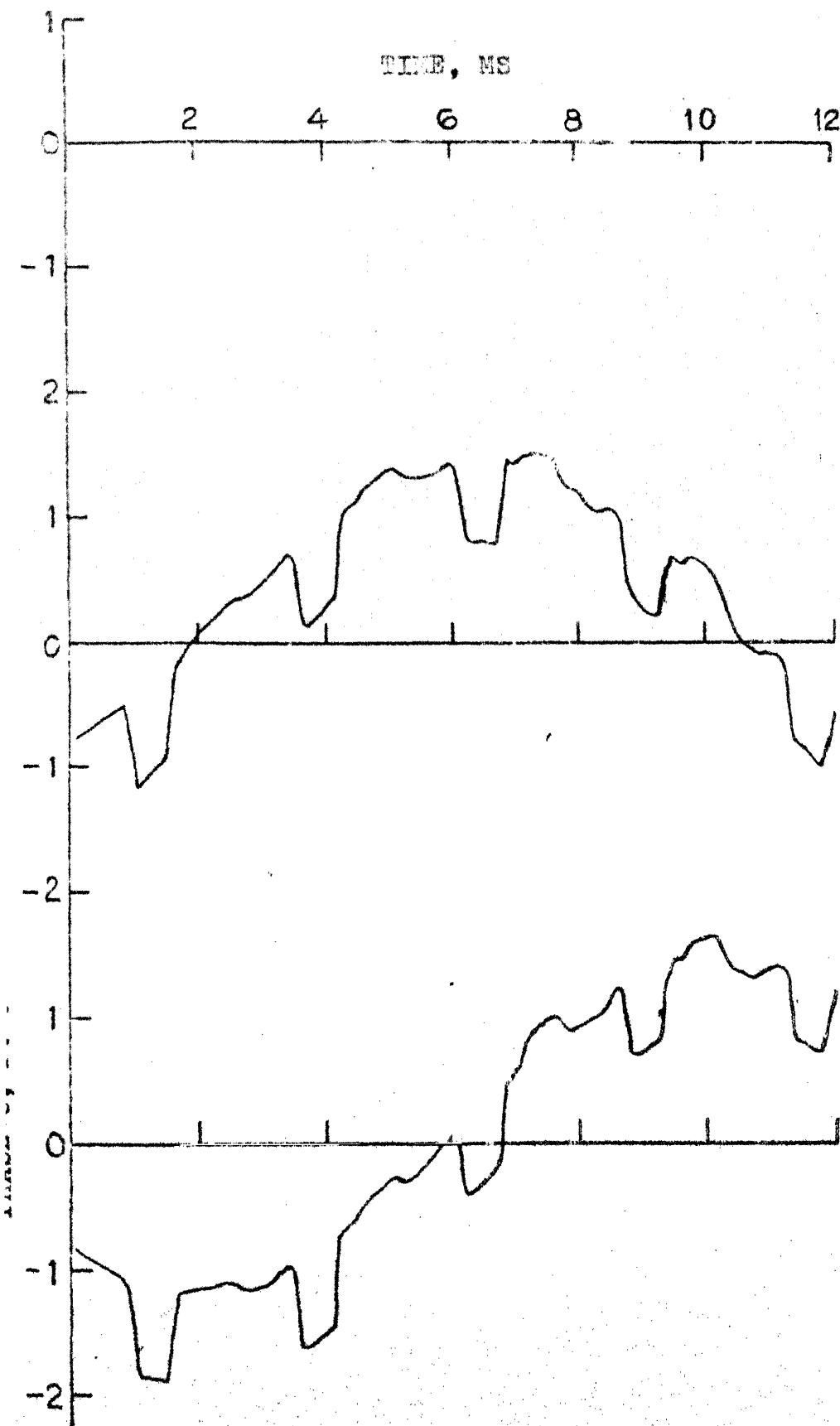
FIGURE 4.2: FAULT POINT VOLTAGE WAVEFORMS OF
BOONYUBOL'S EXAMPLE — FAULT AT

27.94 m m.

S. C. A

13.58m

○ ○ ○
○ × ○ a
○ ○

27.94m.m.

S C A

○ 305 m.m.

19.98

× b

31.22 m

41.55

16.31m

○ ○
○ × ○ c
○ ○

0.88
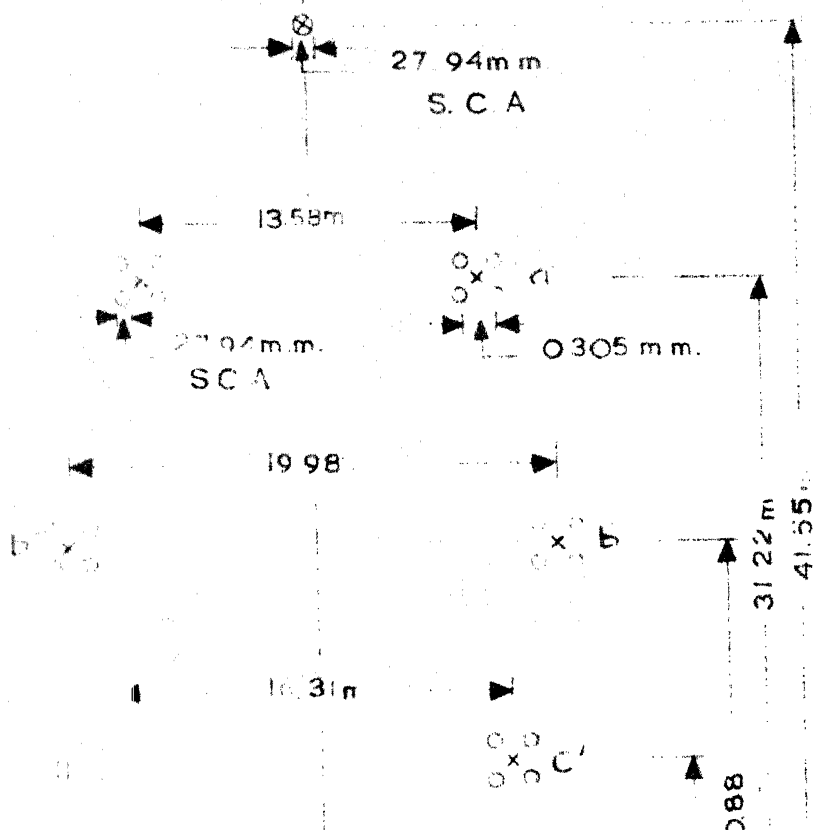
derived from the system short circuit power and voltages at these buses. This simplified model for the source is somewhat approximate. However, here the aim is just to illustrate the application of the modified Fourier transform method of fault transient analysis for a double circuit line and hence source sides are not rigorously represented. For any particular system study, the source representation can be easily modified to suit particular requirements. For the present thesis, more complicated source representation is not considered to be necessary. However, there is no limitation for the applicability of the method for a detailed source side representation. The other lines terminating at the ends of the line under consideration can be represented quite accurately by equivalent shunt elements at these buses, the values of which are obtained by finding out the input admittance offered by these lines at these buses, taking into consideration the distributed nature of these lines and the terminal conditions existing at the remote ends of these lines.

The line parameters are calculated using the Carson's formulae. The effect of the presence of the ground wire is included by making the necessary correction to the line parameter matrices. The 50 Hz line parameters and other system data are given in Appendix D.

4.6 COMPUTATIONAL RESULTS

4.6.1 Fault Overvoltages on Single Circuit and Double Circuit Lines :

The transient due to initiation of a single line to ground fault on the double circuit line, as described in the last section, is studied. In this study, the line is assumed to be ideally transposed and the 50 Hz line parameters are used in the calculations. The fault is considered to be at the mid span on phase 'a'. It is assumed that the fault occurs at the instant when the voltage of phase 'a' at fault point is at positive peak. Next, the presence of the second circuit is ignored and the fault study is conducted for the single circuit line, keeping all the parameters the same. A comparison of the waveforms of the sound phase voltages at fault point for both the cases of single and double circuit lines is made in Figure 4.4. A maximum peak of -1.78 p.u. is observed to occur on phase- c' of the second circuit for the case of the double circuit line representation. For the single circuit line case, the maximum peak voltage of -1.98 p.u. occurs on phase 'c'. The voltage waveforms for the double circuit line case clearly exhibit more high frequency oscillations than those for the single circuit line case. This appears to be due to the fact that there are three distinct modes of propagation with three distinct velocities of wave propagation in the case of transposed double circuit lines; whereas for
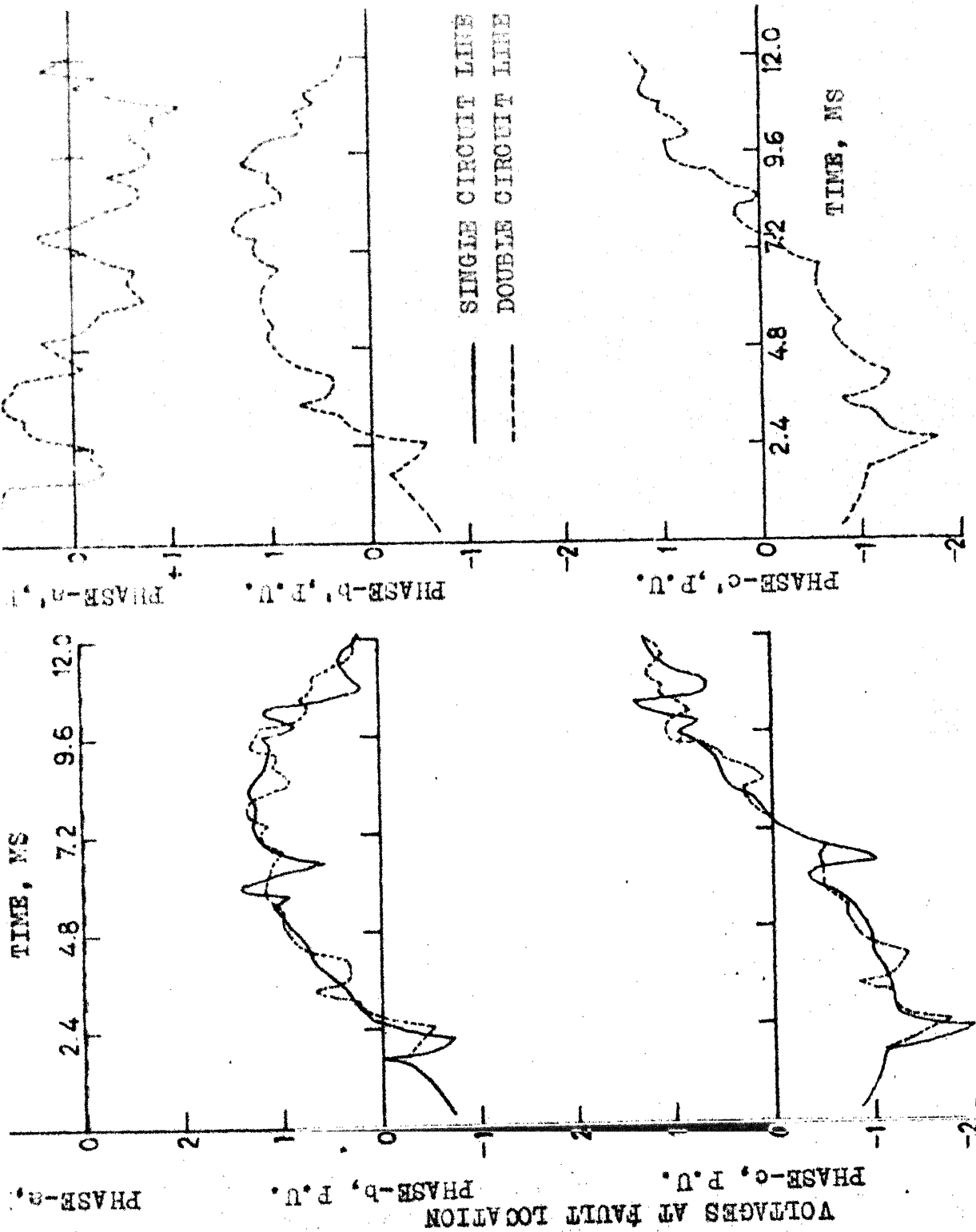
FIGURE 4.4: FAULT TRANSIENT ON TRANSPOSED SINGLE CIRCUIT AND DOUBLE CIRCUIT LINES – FAULT AT MID SPAN ON DOUBLE CIRCUIT LINES

the case of a single circuit line, there are only two such distinct modes and velocities of propagation. Among these three modes for the double circuit line, two are ground modes, corresponding to the sum $([P_a]+[P_b])$ and the difference $([P_a]-[P_b])$ propagation coefficient matrices. The other one corresponds to the aerial mode of the sum and difference propagation coefficient matrices (vide Section 3.2).

4.6.2 Transposed versus Untransposed Line:

The fault study is next conducted for the double circuit line, now assuming the line to be untransposed. For this case, the modal transformation is evaluated at each frequency of the numerical integration range. The line parameters are again assumed to be the 50 Hz values. The phase voltage waveforms at fault point (mid span) for the untransposed case are compared with those of the transposed case in Figure 4.5. It is to be noted that the maximum peak voltage in phase - c' is -1.97 p.u. for the untransposed configuration, whereas the maximum peak for the transposed configuration is -1.78 p.u.

4.6.3 Effect of Incorporating Frequency Dependence of Line Parameters:

Next the continuous variation of line parameters with frequency is included in the calculation and the fault studies are repeated for both the transposed and the untransposed configurations of the line. The voltage

14.4

waveforms for the cases of transposed and untransposed
lines are plotted in Figures 4.6 and 4.7 respectively. In
each case the waveforms with and without incorporation of
frequency dependence of line parameters are shown. The
reduction in the maximum peak voltages when frequency
dependence is included are about 11 percent and 13 percent
for the transposed and untransposed cases respectively.

The maximum peak voltages, obtained in the various
cases for mid span fault on the double circuit line are
summarized in Table 4.1.

Table 4.1

Case	Max.peak voltage p.u.
Transposed line, without frequency dependence	-1.78
Transposed line, with frequency dependence	-1.585
Untransposed line, without frequency dependence	-1.97
Untransposed line, with frequency dependence	-1.72

4.6.4 Overvoltages for Off-Centre Fault Locations:

So far in this chapter, the studies conducted
have been for the mid span fault location. Next the case
of the transposed line is considered and the study is
repeated for different intermediate fault locations,ignoring
the variation of line parameters with frequency. The sound
phase voltage waveforms at fault point for the case of fault
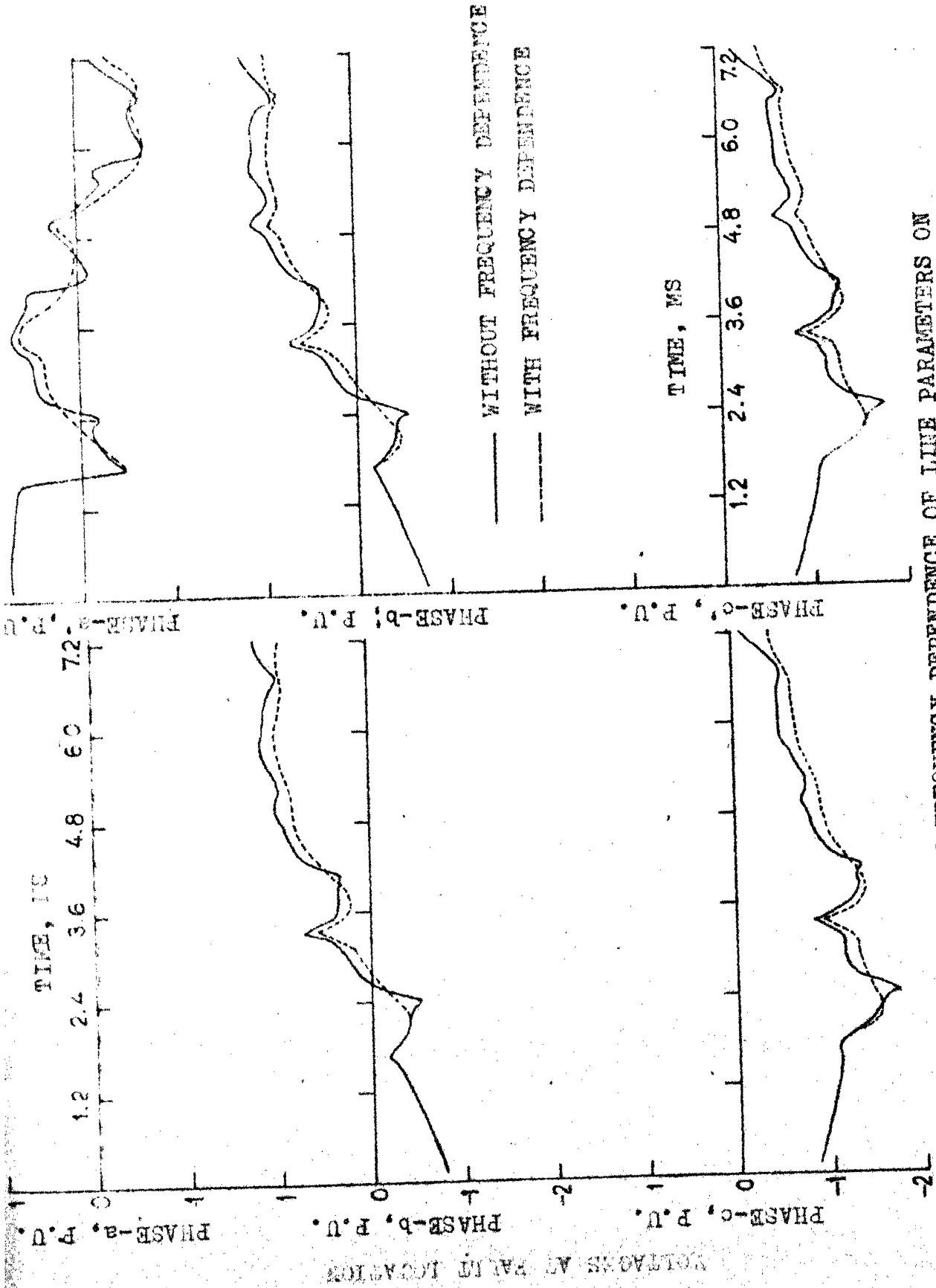
FIGURE 4.6: EFFECT OF FREQUENCY DEPENDENCE OF LINE PARAMETERS ON
FAULT TRANSIENTS – TRANSPOSED LINE – FAULT AT MID SPAN
ON PHASE-a.

at a distance of 120 kms from one end of the line are plotted in Figure 4.8. The maximum peak voltages for various locations of fault points are given in Table 4.2. It is to be noted that the transients for fault locations symmetric with respect to the mid point of the line are identical because of the symmetrical nature of the system considered.

Table 4.2

Distance of the fault point from the line end in kms. (Total line length considered is 480 kms.)	Maximum peak voltage in p.u.
240	-1.78
180	-1.57
120	-1.55
60	-1.40
8	-1.36

The above off-centre fault location studies indicate that the maximum overvoltage occurs in the case of the mid span fault.

In all the above studies, the fault is assumed to occur only on the bottom most conductor (Figure 4.3) at the instant when the voltage of that phase at that point is passing through its positive peak. For the case of untransposed lines, if the fault occurs on one of the other two phase conductors, the fault transients and the associated maximum peaks will be
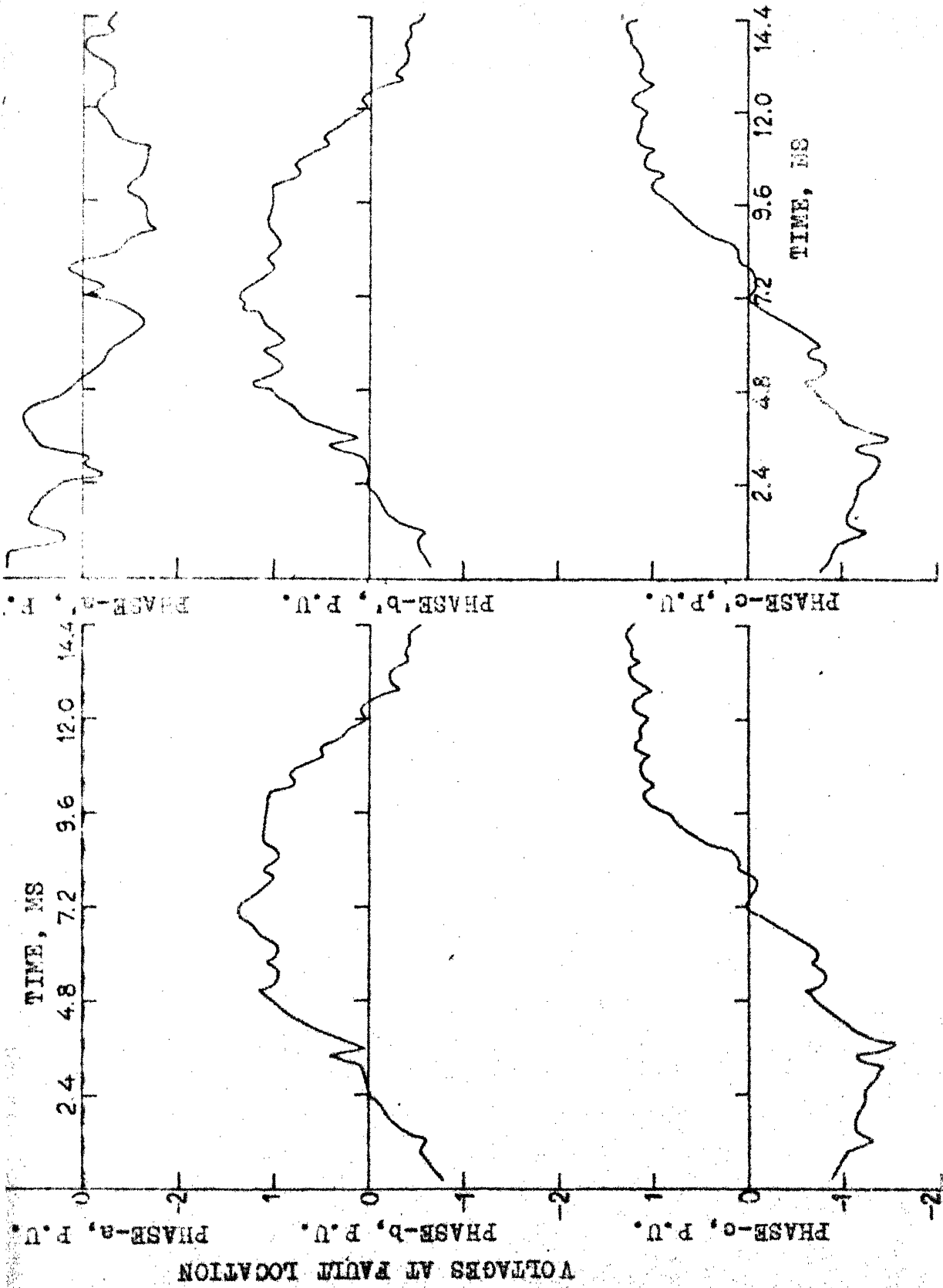
FIGURE 4.8: EFFECT OF CHANGE IN FAULT LOCATION TO QUARTER SPAN – TRANSPOSED LINE – WITHOUT FREQUENCY DEPENDENCE.

different. However, there is no restriction on the
applicability of the developed algorithm for studying the
transients due to fault on the other phase conductors. As
the aim of the thesis is to present a method of analysis
using the modified Fourier transform method for double
circuit lines, rather than any particular system study,
consideration of various cases of data permutation have
not been studied. It may be pointed out here that the
case of considering the fault on the bottom-most conductor
need not be the most serious from the point of view of
determining the system insulation requirements. Such
considerations are outside the scope of this thesis, but
are essential in carrying out actual system studies using
the method proposed in this thesis or using any other
method of analysis. This comment also applies for other
examples that are considered in subsequent chapters.

4.7 CONCLUSIONS

The transient due to initiation of a single line
to ground fault on a double circuit transmission line has
been studied in this chapter. A mathematical analysis of
the overvoltage transients, developed on the sound phases
following the fault initiation at any intermediate point
on the line has been presented. The double circuit line
has been accurately modelled for the study, taking into
account the mutual coupling between the two circuits.
The study makes use of the modified Fourier transform method.

Both the cases of transposed and untransposed configurations of the line have been considered. As a frequency independent modal transformation exists for the case of the transposed double circuit line, as shown in Chapter 3, this case involves much less computational effort. In the case of the untransposed line, the evaluation of the modal transformation matrix at each frequency in the numerical integration range is required. The maximum peak voltage in the case of the untransposed line is observed to be nearly 10 percent higher than that of the transposed case.

The effect of frequency dependence of line parameters on the fault transient overvoltages for both the transposed and untransposed lines is investigated,using Carson's formulae for the calculation of line parameters. The inclusion of frequency dependence of line parameters is observed to reduce the maximum peak voltages by nearly 11 percent and 13 percent for the cases of transposed and untransposed lines respectively.

Finally the fault location is changed to various off-centre points along the line and the fault transients are studied. The mid span fault is found to give rise to the maximum peak overvoltage.

CHAPTER 5

ENERGISING TRANSIENT ON A DOUBLE CIRCUIT
TRANSMISSION LINE

5.1 INTRODUCTION

An EHV transmission line is subjected to high over-
voltages because of energisation and re-energisation of the
line. An accurate evaluation of these transients is
essential for the economic design of the insulation. A
double circuit line being a common element of any power
system, a study of the transients due to its energisation
is of importance. In the existing literature, the double
circuit line transients have been analysed mainly using the
transient network analyser [26]. In this chapter, the
application of the modified Fourier transform method for
analysing these transients and the results of these studies
for both transposed and untransposed configurations of a
typical line are presented.

5.2 PROBLEM DESCRIPTION

In practice, the energisation of a double circuit
line is done in two stages. First one circuit is energised
and after an elapse of time, the second circuit is closed.
There is bound to be a delay of a few seconds at least if
not a few minutes between the closing of the first and
second circuits of a double circuit line. During this delay

period, the transient arising in the first circuit due to its energisation would have died down and this circuit would be in steady state. Thus essentially two transients are involved in the study of the energisation phenomenon of a double circuit line, viz. (i) the transient due to the closing of the first circuit, while the second circuit is unenergised (ii) the transient due to the closing of the second circuit when the first circuit is already in steady state.

5.3 FIRST CIRCUIT ENERGISATION

In this section, a mathematical development of the analysis of transient due to the first circuit closure of a double circuit line, open at the receiving end is described.

As described in the earlier chapters, the general expressions for the transformed voltage and current vectors at any intermediate point on a double circuit transmission line at a distance x units away from the sending end are given by the expressions

$$\underline{V} = [\text{Cosh } \psi x] \, \underline{K} + [\text{Sinh } \psi x] \, \underline{M}$$

$$\underline{I} = -[Z]^{-1}[\psi] \, \{[\text{Cosh } \psi x] \, \underline{M} + [\text{Sinh } \psi x] \, \underline{K}\}$$

where $\underline{K}$ and $\underline{M}$ are constant vectors, which are determined by the terminal conditions of the line and $[\psi]$, $[Z]$, $[\text{Cosh } \psi x]$ and $[\text{Sinh } \psi x]$ are the same as defined in the earlier chapters.

If the voltage transforms at the sending and receiving ends of the line ($\underline{V}_S$ and $\underline{V}_R$) are known, these conditions when forced on the general expressions determine the constant vectors $\underline{K}$ and $\underline{M}$ as follows:

$$\underline{K} = \underline{V}_S$$

$$\underline{M} = [\text{Cosech } \psi \ell] \, \underline{V}_R - [\text{Coth } \psi \ell] \, \underline{V}_S$$

where ℓ is the length of the line.

Hence the sending and receiving end transformed currents can be expressed in terms of the transformed voltages at the two ends as

$$\begin{bmatrix} \underline{I}_S \\ \underline{I}_R \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [A] \end{bmatrix} \begin{bmatrix} \underline{V}_S \\ \underline{V}_R \end{bmatrix} \tag{5.1}$$

where $[A] = [Z]^{-1}[\psi][\text{Coth } \psi \ell]$

and $[B] = -[Z]^{-1}[\psi][\text{Cosech } \psi \ell]$.

In the above expression $\begin{bmatrix} [A] & [B] \\ [B] & [A] \end{bmatrix}$ can be interpreted as the short circuit admittance parameter matrix of the transmission line, treated as a two port network.

Now considering a double circuit line, which is connected to a voltage source with source resistance R_S and source inductance L_S at the sending end and whose receiving end is open (Figure 5.1.1), the voltage source with source impedance at the sending end can be replaced
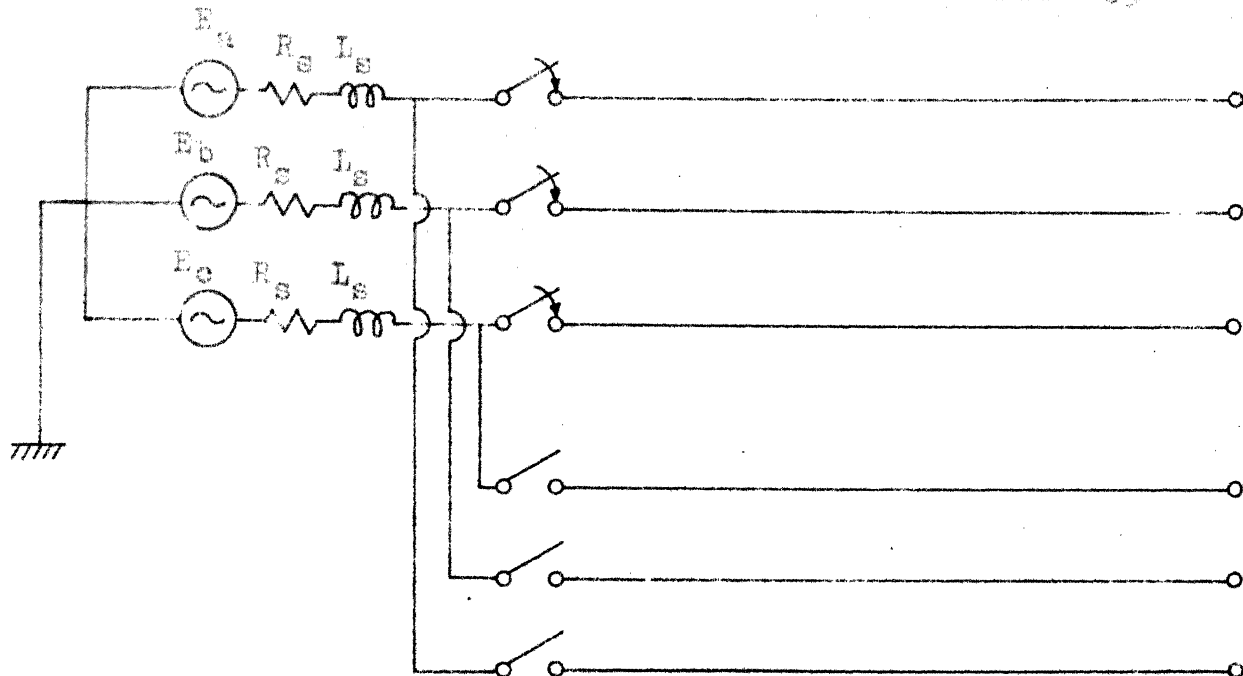
FIGURE 5.1.1: SYSTEM DIAGRAM WITH SOURCE REPRESENTATION

$$I_{s1} = \frac{E_a}{Z_{s1}}$$

$$I_{s2} = \frac{E_b}{Z_{s2}}$$

$$I_{s3} = \frac{E_c}{Z_{s3}}$$
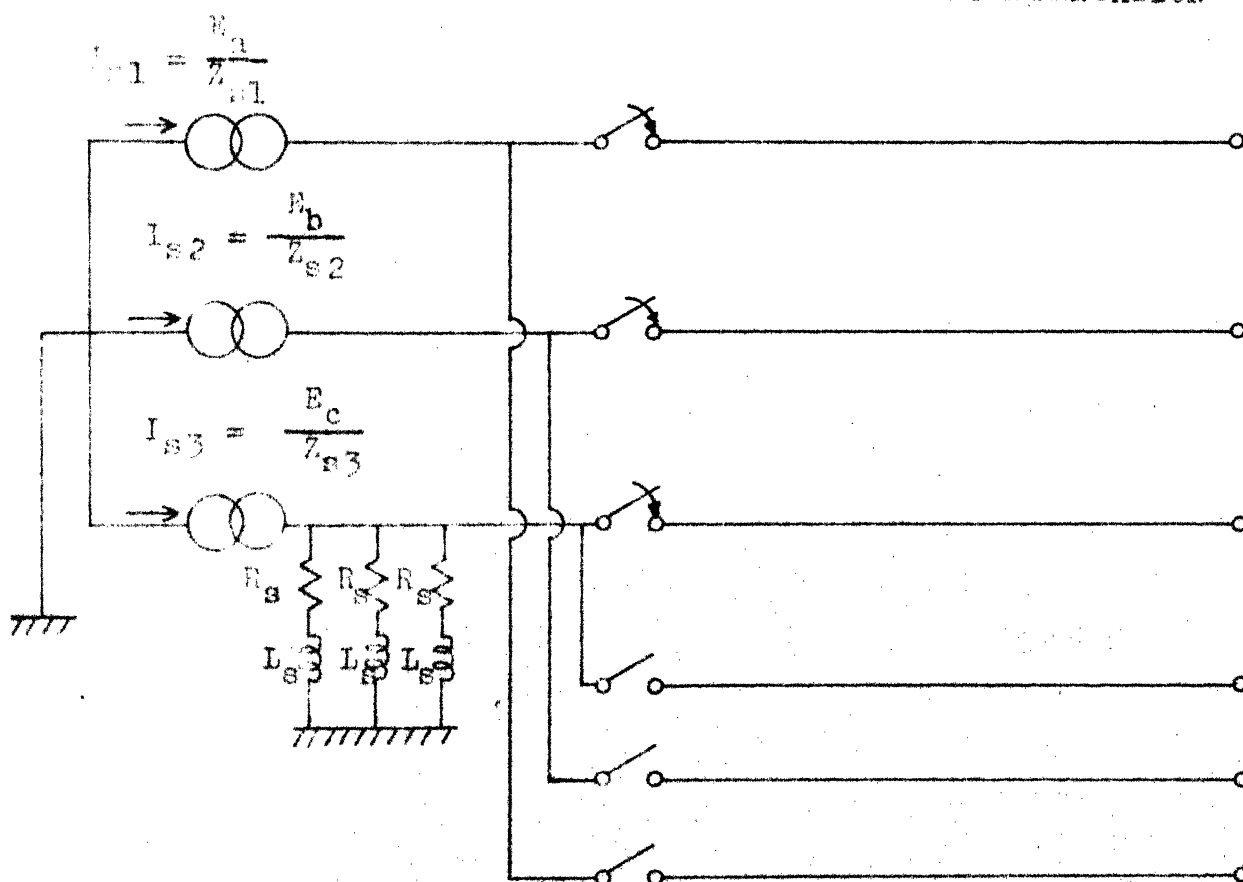
FIGURE 5.1.2: SYSTEM DIAGRAM WITH EQUIVALENT
CURRENT SOURCE.

by an equivalent source with shunt admittance as shown in Figure 5.1.2.

As the second circuit of the double circuit line remains unenergised for the case under consideration, the forcing function is converted as a current source so that when the current injected to the second circuit phases at the sending end are taken as zero, the problem is directly simulated.

Now the double circuit line with the source admittance at the sending end can be viewed as a two port network and the incorporation of the shunt admittance at the sending end just introduces this additional admittance to the self admittance term of the sending end port of the short circuit admittance parameter matrix of the line. Therefore for this case, the two-port admittance equation takes the form

$$
\begin{bmatrix} \underline{I}_s \\ \underline{0} \end{bmatrix} = \begin{bmatrix} ([A]+[Y_s]) & [B] \\ [B] & [A] \end{bmatrix} \begin{bmatrix} \underline{V}_s \\ \underline{V}_R \end{bmatrix} \tag{5.2}
$$

where [A] and [B] are as defined in equation (5.1) except that they are now (6x6) matrices as we are dealing with the double circuit line.

Here $\quad \underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$[Y_S] = \begin{bmatrix} 1/[R_S+(a+j\omega)L_S] & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/[R_S+(a+j\omega)L_S] & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/[R_S+(a+j\omega)L_S] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\underline{I}_S = \begin{bmatrix} E_a/[R_S+(a+j\omega)L_S] \\ E_b/[R_S+(a+j\omega)L_S] \\ E_c/[R_S+(a+j\omega)L_S] \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Equation (5.2) can be manipulated to write $\underline{V}_R$ and $\underline{V}_S$ in terms of $\underline{I}_S$ as given below.

$$\underline{V}_R = \{[B]-([A]+[Y_S])[B]^{-1}[A]\}^{-1} \underline{I}_S$$

$$\underline{V}_S = \{[A]+[Y_S]-[B][A]^{-1}[B]\}^{-1} \underline{I}_S$$

The calculation of matrices $[\psi]$, $[\text{Cosech}\,\psi\ell]$, $[\coth\psi\ell]$ and hence the calculation of matrices $[A]$ and $[B]$ differ for the transposed and untransposed configurations of the double circuit line. It has been shown in Chapter 3 that because of the special structure of the propagation coefficient

matrix for the case of the transposed double circuit line, a frequency independent constant modal transformation matrix exists for this case. This very much simplifies the calculation procedure for the transposed configuration. For handling the untransposed double circuit line, the eigenvalue eigenvector analysis of propagation coefficient matrix is to be performed at each frequency to arrive at the modal transformation matrix at that frequency. The frequency dependence of the line parameters can be directly incorporated in the analysis by using the appropriate line parameter values at each frequency, calculated using the Carson's formulae.

5.4 COMPUTATIONAL RESULTS

The results of the study of single circuit energisation of a typical 400 KV double circuit line, open at the receiving end are presented in this section. The configuration and parameters of the line are the same as for the example considered in Chapter 4 (Appendix D) for fault studies. The length of the line is 240 kms and the source is taken as purely inductive of value 0.1 Henry/phase. The effect of the presence of the earth wire is included by making the necessary correction to the phase parameter matrices of the line. The earth resistivity is assumed to be 100 ohm-meter.

5.4.1 Transposed Line Versus Untransposed Line:

The phase voltage waveforms at the open receiving
end of the first circuit for the transposed and untransposed
cases are compared in Figure 5.2. For both the cases, the
energisation has been done when phase 'a' source voltage is
at positive peak. From the plots, it is observed that a
maximum peak voltage of -2.45 p.u. occurs on phase 'a' at
9.2 m.secs. for the case of transposed configuration. The
maximum peak is 2.24 p.u. for the case of untransposed
configuration and this occurs on phase 'c' at 12.8 m.secs.
Because of the low order of magnitude of induced voltages
on the unenergised second circuit, they have not been plotted.

5.4.2 Single Circuit Versus Double Circuit Line
      Representations:

Next a set of energising studies are conducted,
neglecting the presence of the second circuit with a view
of comparing these results with those of double circuit
representation.

For the case of transposed line, there is no appreci-
able difference in the waveshapes of receiving end voltages
for the single circuit and double circuit representations.
The reason for this appears to be that the induced charges
on the three phases of the unenergised second circuit
cancel with one another even during the transient period
in the case of the transposed double circuit line.

TIME, MS

——————— TRANSPOSED
– – – – – UNTRANSPOSED
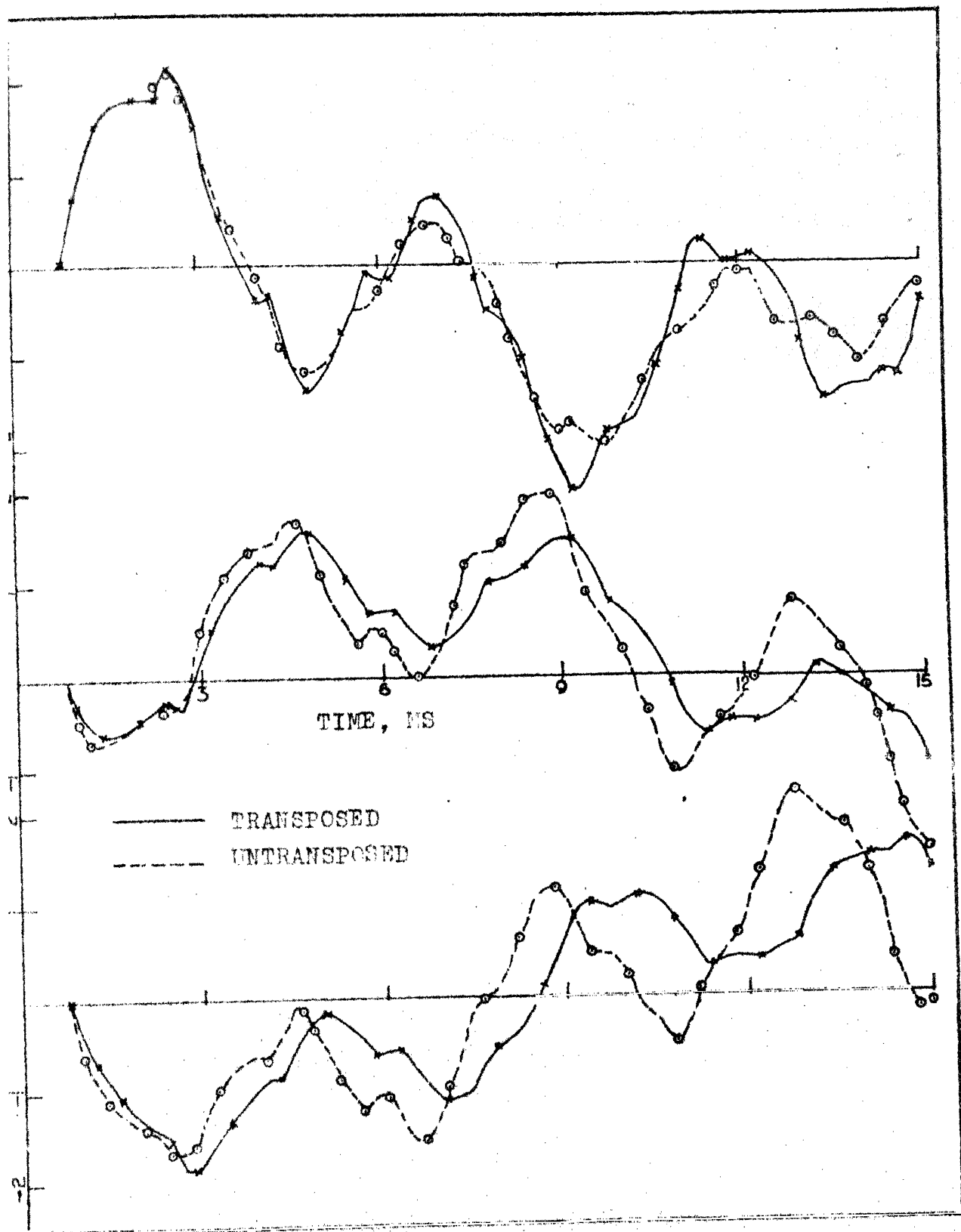
FIGURE 5.2: VOLTAGE TRANSIENTS AT RECEIVING END – FIRST CIRCUIT
ENERGISATION – SECOND CIRCUIT OPEN.

The voltage waveforms at the open receiving end for the untransposed single and double circuit line representations are compared in Figure 5.3. From the plots, it can be observed that the maximum peak voltage of 2.22 p.u. occurs on phase 'c' at 13.26 m.secs. for the case of single circuit representation; whereas a maximum peak of 2.24 p.u. occurs on phase 'c' at 12.8 m.secs. for the case of double circuit representation.

5.5 SIMULATION OF SECOND CIRCUIT ENERGISATION

In this section, the analysis of the transient due to the closing of the second circuit, when the first circuit is in steady state, is described. Again the same example of a double circuit line, open at the receiving end and connected to a voltage source with source impedance at the sending end is considered and an algorithm for the analysis of the second circuit energising transient, using the modified Fourier transform method is developed.

The closing of the second circuit is simulated by voltage cancellation technique and superposition. The steps involved can be described with the help of Figures 5.4.1 and 5.4.2 as follows:

(i) First assuming that the second circuit continues to be unenergised, the response at any point of interest on the line (particularly the voltage at the open receiving end) and the voltages across the circuit breaker terminals
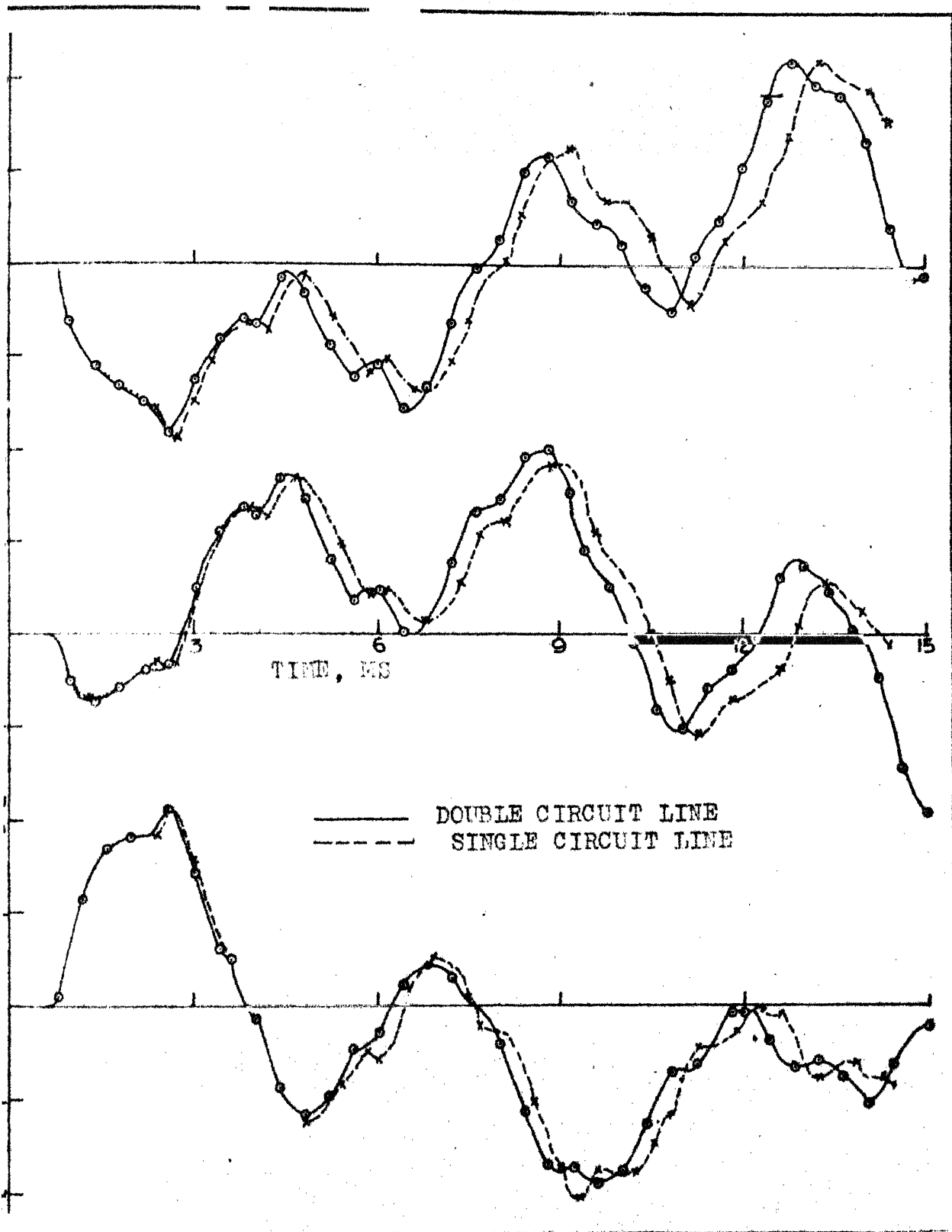
FIGURE 5.3: RECEIVING END VOLTAGE WAVEFORMS FOR UNTRANSPOSED
DOUBLE AND SINGLE CIRCUIT LINES - FIRST CIRCUIT
ENERGISATION - SECOND CIRCUIT OPEN.
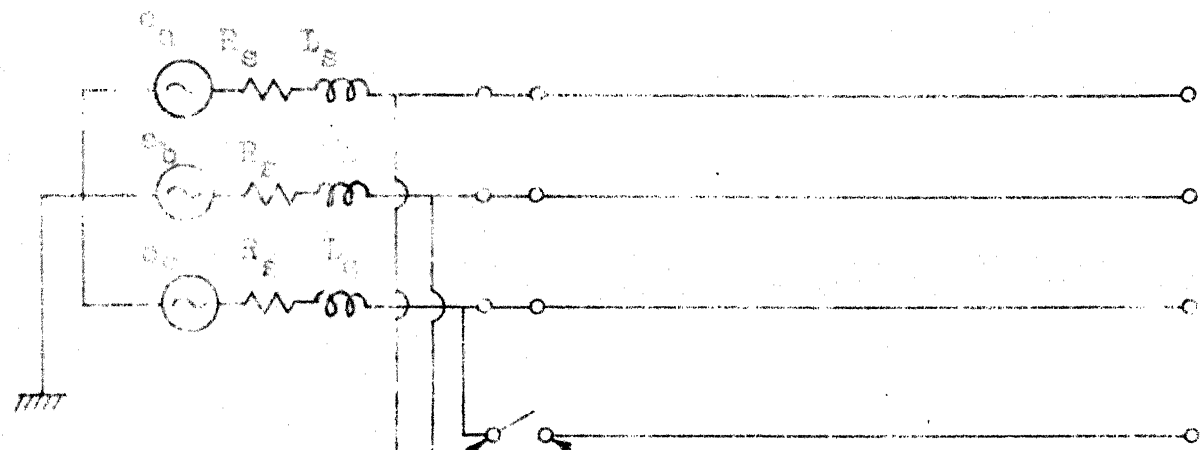
of all the three phases at the sending end of the second
circuit are calculated. At this stage, as the first circuit
is already in steady state, this step of the calculation is
a steady state analysis.

(ii) The voltages across the breaker poles, calculated
in step (i) are injected as cancellation voltages, with
reversed polarity, across the same poles. The sending
end terminals of the first circuit are short circuited to
ground. The response at the above mentioned point of
interest on the line is again calculated, now for these
injected cancellation voltages alone.

(iii) Superimposing these two component responses, the
total response is obtained.

Therefore, the procedure adopted for calculating
the second circuit energising transient of a double
circuit line essentially boils down to a steady state
analysis, keeping the second circuit unenergised and
finding the transient response of the line for a known set
of sending end voltage conditions.

5.5.1 Calculation of Steady State Response:

The method of calculating the steady state response
at any point of interest on the line and the voltages across
the circuit breaker poles at the sending end of the second
circuit (step i of last section) is very similar to the

analysis of the transient subsequent to the closing of the first circuit when the second circuit is unenergised (described in Section 5.3). The only difference is that now the transmission line equations become full differential equations and the variables to be solved for are voltage phasors instead of instantaneous values of voltages because of the steady state nature of the analysis. It is to be noted that now all the variables involved are phasors at source frequency and no other frequency is involved in the calculations.

By adopting a similar solution procedure as in Section 5.3, the expressions for the sending end and receiving end voltage phasors in terms of sending end current phasors can be derived as follows:

$$\underline{V}_S = ([A]+[Y_S]-[B][A]^{-1}[B])^{-1} \, \underline{I}_S$$

$$\underline{V}_R \doteq \{ [B]-([A]+[Y_S])[B]^{-1}[A] \}^{-1} \, \underline{I}_S$$

where $[A] = [Z]^{-1}[\psi][\text{Coth } \psi\ell]$

$[B] = -[Z]^{-1}[\psi][\text{Cosech } \psi\ell]$

ℓ = length of the line

$[Z] = [R]+j\,\omega_S[L]$

$[\psi] = [Q][\gamma][Q]^{-1}$

$[\gamma]$ = diagonal matrix, whose diagonal elements are the square roots of the eigenvalues of the matrix product $[Z][Y]$

$$[Y] = j\omega_s[C]$$

ω_s = angular velocity corresponding to source frequency

$[Q]$ = eigenvector matrix of the matrix product $[Z][Y]$

$$[\text{Coth}\psi\ell] = [Q][\text{Coth }\gamma\ell][Q]^{-1}$$

$$[\text{Cosech}\psi\ell] = [Q][\text{Cosech }\gamma\ell][Q]^{-1}$$

$$[Y_s] = \begin{bmatrix} 1/(R_s+j\omega_sL_s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/(R_s+j\omega_sL_s) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/(R_s+j\omega_sL_s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R_s = resistance of the source/phase

L_s = inductance of the source/phase

$[R]$ = resistance/unit length matrix of the line

$[L]$ = inductance/unit length matrix of the line

$[C]$ = capacitance/unit length matrix of the line,

and
$$\underline{I}_s = \begin{bmatrix} 1/(R_s+j\omega_sL_s) \\ (-0.5+j0.866)/(R_s+j\omega_sL_s) \\ (-0.5-j0.866)/(R_s+j\omega_sL_s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If the double circuit line is transposed, the matrix [Q] takes a simple form as shown in Chapter 3. For the case of untransposed double circuit line, the eigenvalue eigenvector analysis is to be performed for the matrix product [Z][Y] to obtain the matrices $[\gamma]$, [Q] and $[Q]^{-1}$.

When the above analysis is performed for the double circuit line example under study in this chapter, the steady state induced voltages on the unenergised second circuit at the sending and receiving ends are calculated to be as follows:

Transposed case:

At receiving end - Phase-a' $(-0.83 \times 10^{-6} - j0.11 \times 10^{-5})$ p.u.

Phase-b' $(0.19 \times 10^{-7} - j0.72 \times 10^{-5})$ p.u.

Phase-c' $(-0.97 \times 10^{-7} + j0.19 \times 10^{-5})$ p.u.

At sending end - Phase-a' $(-0.28 \times 10^{-5} + j0.77 \times 10^{-6})$ p.u.

Phase-b' $(0.68 \times 10^{-6} - j0.29 \times 10^{-5})$ p.u.

Phase-c' $(-0.22 \times 10^{-5} + j0.80 \times 10^{-5})$ p.u.

Untransposed case:

At receiving end - Phase-a' $(0.016 + j0.68 \times 10^{-3})$ p.u.

Phase-b' $(-0.025 - j0.025)$ p.u.

Phase-c' $(-0.082 - j0.079)$ p.u.

At sending end - Phase-a' $(0.011 + j0.40 \times 10^{-5})$ p.u.

Phase-b' $(-0.029 - j0.025)$ p.u.

Phase-c' $(-0.083 - j0.077)$ p.u.

The steady state voltages on the first circuit at the sending and receiving ends are calculated to be 1.03 p.u. and 1.07 p.u. respectively.

From the above analysis, it can be observed that the order of magnitude of the induced voltages on the second circuit is small when compared to the corresponding first circuit voltages. Hence it can be assumed that no voltages are induced on the second circuit conductors during this step of the calculation. In that case, the voltages across the sending end breaker poles of the second circuit can be taken as the steady state voltages at the sending end of the connected phases of the first circuit in step (i) of Section 5.5. However, if more accuracy is required, the induced voltages on the second circuit can be taken into account while calculating the voltages across the breaker poles of the second circuit at the sending end for the case of the untransposed line.

5.5.2 Calculation of Transient Response for the Injected Voltages:

The step (ii) of this calculation involves evaluation of transient response of the transmission line for known sending end voltage conditions (viz. the first circuit terminals connected to ground and the injected voltages connected to the second circuit) with the receiving end terminals kept open.

These conditions, when substituted in the general solution expressions for transformed voltage and current vectors at any point on the transmission line lead to the evaluation of the constant vectors $\underline{K}$ and $\underline{M}$ as given below:

$$\underline{K} = \underline{V}_S$$

$$[\text{Cosh}\psi\ell]\ \underline{M} + [\text{Sinh}\psi\ell]\ \underline{V}_S = 0$$

i.e.
$$\underline{M} = -[\text{Tanh}\psi\ell]\ \underline{V}_S$$

Therefore the transformed receiving end voltage vector becomes

$$\underline{V}_R = [\text{Cosh}\psi\ell]\ \underline{V}_S - [\text{Sinh}\ \psi\ell][\text{Tanh}\psi\ell]\ \underline{V}_S$$

$$= [\text{Sech}\ \psi\ell]\ \underline{V}_S$$

where $[\text{Sech}\ \psi\ell] = [Q][\text{Sech}\ \gamma\ell][Q]^{-1}$

and $[\gamma]$ and $[Q]$ are the eigenvalue and eigenvector matrices of the propagation coefficient matrix of the line respectively.

The inverse transformation of the transient response to the time domain is done by performing the numerical integration as described earlier in Section 2.2.

5.6 COMPUTATIONAL RESULTS

The same 400 KV double circuit line example, considered earlier in Section 5.4 of this chapter, is taken up to study the transient due to second circuit closure and the results are discussed below.

5.6.1 Transposed Versus Untransposed Configurations:

The above study is done for both the cases of transposed and untransposed configurations of the line. Again the existence of the frequency independent modal transformation matrix for the transposed double circuit line makes the computation for this case much easier when compared to the untransposed case. The comparison of the voltage waveforms at the open receiving end of the second circuit for the transposed and untransposed cases is done in Figure 5.5. For both the cases the closing of the circuit breakers is assumed to be at the instant when the source voltage of phase-a' is at positive peak. It is observed from the plots that a maximum peak voltage of 2.02 p.u. occurs on phase-a' at 1.02 m.secs. for the case of transposed line, whereas a maximum peak of 2.23 p.u. occurs on phase-a' at 0.9 m.secs. for the case of untransposed line. The receiving end voltages on first circuit subsequent to the closure of the second circuit are not plotted because of the lower magnitudes of these voltages in comparison with those of the second circuit phases.

5.6.2 Single Versus Double Circuit Line Representations:

Next a balanced three phase voltage of 1.03 p.u. (the steady state phase voltages at the sending end terminals of the first circuit before the closure of the second circuit) is applied to the second circuit and the resulting transient overvoltages at the open receiving end
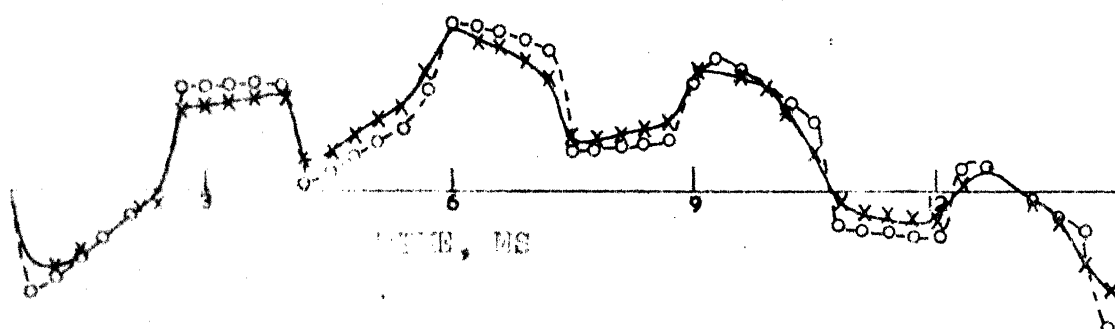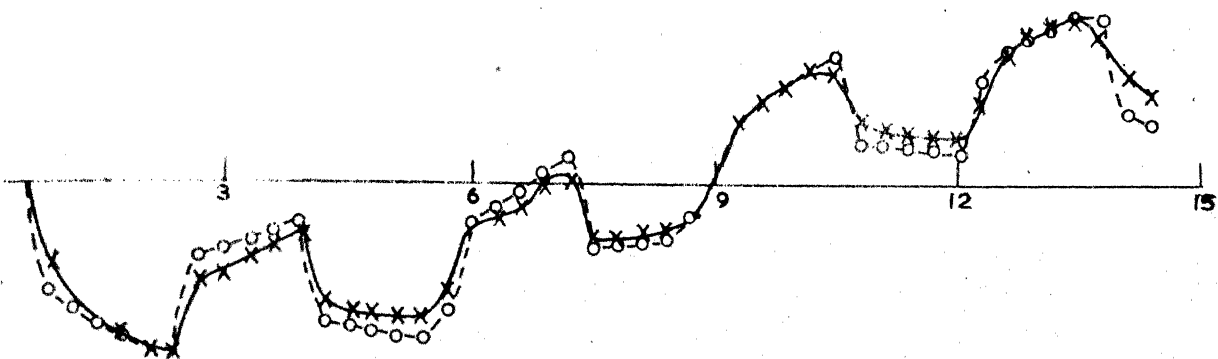
UNTRANSPOSED
TRANSPOSED

are computed, neglecting the presence of the first circuit. This is done to compare the transients for the single and double circuit representations of the line.

For the case of transposed configuration of the line, there is no appreciable difference in the open receiving end voltage waveforms for single and double circuit representations since the induced voltages on the second circuit due to the currents in the three phases of the first circuit balance one another. However, it is observed later in Section 7.7.1 that once the method is extended for handling the nonsimultaneous closing of the poles of the breakers, even for the transposed configuration of the line, the double circuit representation gives rise to a difference in voltage waveforms as compared to the single circuit representation.

For the case of the untransposed line, a considerable difference in the waveshapes of the receiving end voltage waveforms is observed even for simultaneous closing of the breaker poles of the second circuit. A maximum peak over-voltage of magnitude -2.5 p.u. is developed on phase-a' at 12 m.secs. for double circuit representation. On the other hand, a maximum peak of 2.02 p.u. occurs on phase-a' at 1.26 m.secs. for the case of single circuit representation. Apart from the difference in peak values, the voltage waveforms (Figure 5.6) for the single and double circuit representations
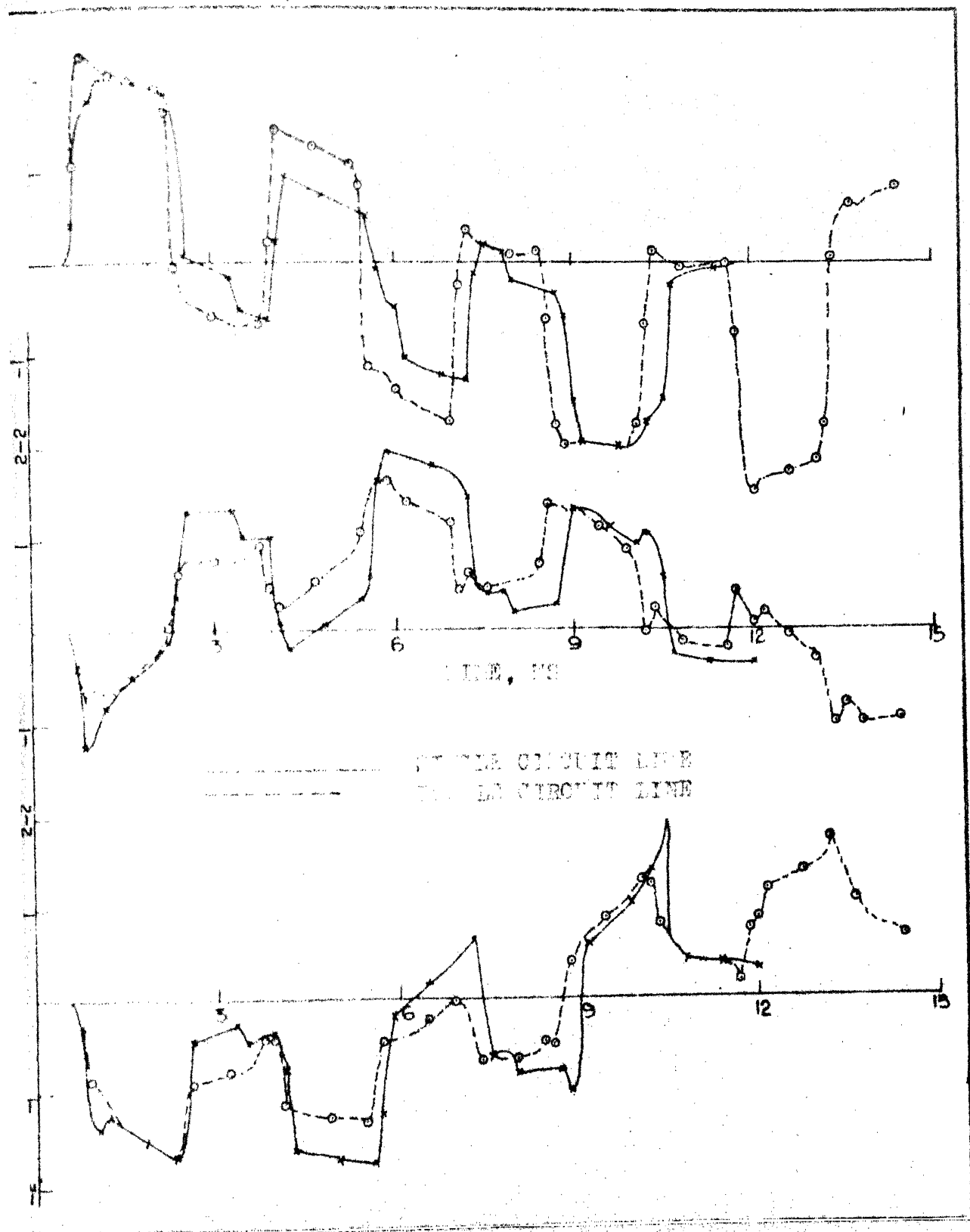
FIGURE 5.6: RECEIVING END VOLTAGE WAVEFORMS FOR UNTRANSPOSED
DOUBLE AND SINGLE CIRCUIT LINES — SECOND CIRCUIT
ENERGISATION — FIRST CIRCUIT IN STEADY STATE.

differ widely in shape also. This seems to be due to the
fact that for the case of untransposed double circuit lines,
there are six distinct modes and velocities of propagation;
whereas for untransposed single circuit lines, there are
only three such distinct modes and velocities of propagation.

5.7 EFFECT OF INCORPORATION OF FREQUENCY DEPENDENCE OF LINE PARAMETERS

Next the continuous variation of line parameter
values with frequency is incorporated in the program and the
voltage transients are computed for the various cases. How-
ever, it has been observed that in all these cases, the
effect of incorporation of frequency dependence of the line
parameters on the voltage waveforms is not substantial.
This appears to be due to the absence of the zero sequence
ground currents in all these cases because of the simultane-
ous closing of the circuit breaker poles and only zero
sequence components of the resistance and inductance of the
line vary considerably with frequency. It is observed
later in Section 7.7.2 that when nonsimultaneous closing
of the circuit breaker poles is incorporated in the program,
the frequency dependence of line parameters affects the
transient voltage waveforms considerably.

5.8 CONCLUSIONS

The modified Fourier transform method has been
applied to analyse the energising transients of a double
circuit line in this chapter. Algorithms have been

developed for computing the transient due to the closing of the first circuit with the second circuit unenergised and that caused by the closing of the second circuit when the first circuit is in steady state. An example of a typical 400 KV double circuit line has been used to evaluate the transients for both transposed and untransposed configurations of the line. The energising transients for the double circuit representation of the line have been compared with the transients caused by the energisation of the corresponding single circuit line (neglecting the presence of the other circuit). The effect of the inclusion of frequency dependence of line parameters on the energising transients is observed to be insignificant for the case of simultaneous closing of the circuit breaker poles.

CHAPTER 6

EXTENSION OF URAM AND MILLER'S METHOD FOR DOUBLE CIRCUIT LINE ENERGISATION TRANSIENT ANALYSIS

6.1 INTRODUCTION

In this chapter, the theory of the Uram and Miller's method [4,5] of calculating the switching transients on transmission lines is extended for the case of double circuit lines. The existence of a constant modal transformation matrix for the case of transposed double circuit lines, as shown in Chapter 3 makes this possible. The example, tried in Chapter 5 (using the modified Fourier transform method) is taken up and the two transients, involved in the energisation of a double circuit line, viz., the transient due to the closing of the first circuit when the second circuit is unenergised and that due to the closing of the second circuit, when the first circuit is in steady state are studied. The results obtained by both the methods are compared. As the Uram and Miller's method is a time domain method, the frequency dependence of the line parameters cannot be directly incorporated in this method. The untransposed lines also cannot be handled by this method. However, it takes relatively less computer time.

6.2 EXTENSION OF URAM AND MILLER'S METHOD FOR DOUBLE CIRCUIT LINE TRANSIENTS

A brief outline of the Uram and Miller's method of solving electrical transients on transmission lines is given in Appendix E. In this section the theory is extended for the case of double circuit lines.

The energisation of a double circuit line is to be studied in two stages. One transient is due to the closing of the first circuit, with the second circuit unenergised. The other possibility is that one circuit may be operating in steady state and the second circuit may be energised. The evaluation of both these transients is essential to arrive at the insulation level of the line and other equipments.

It has been shown in Chapter 3 that a frequency independent constant modal transformation matrix exists for a transposed double circuit line. This makes the extension of the theory of Uram and Miller's method (described in Appendix B) possible for studying both the transients, involved in the case of a double circuit line.

The equations (E.10), (E.11), (E.12) and (E.13) developed for the sending and receiving end voltage and current vectors in the Appendix E hold good for the double circuit line also. But now, the line parameters and hence [Z] and [Y] become 6x6 matrices. The vectors $\underline{k}_1(t)$, $\underline{k}_2(t)$, $\underline{a}(t)$ and $\underline{b}(t)$ are all of order 6.

The modal transformation matrix $[T]$ takes the form

$$[T] = \begin{bmatrix} [Q] & -[Q] \\ [Q] & [Q] \end{bmatrix} .$$

where

$$[Q] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

is the modal transformation matrix for a single circuit three phase line as described in Section 3.2.1.

As described in Chapter 3, the line parameter matrices of the transposed double circuit line have the following special structures:

$$[R] = \begin{bmatrix} [R_a] & [R_b] \\ [R_b] & [R_a] \end{bmatrix} \quad ; \quad [L] = \begin{bmatrix} [L_a] & [L_b] \\ [L_b] & [L_a] \end{bmatrix} \quad ;$$

$$[C] = \begin{bmatrix} [C_a] & [C_b] \\ [C_b] & [C_a] \end{bmatrix}$$

In the above matrices, each of the submatrices, $[R_a]$, $[L_a]$ and $[C_a]$ is a 3x3 matrix, the diagonal elements of which are equal and the off-diagonal elements of which are also equal. Each of the submatrices $[R_b]$, $[L_b]$ and $[C_b]$ is a 3x3 matrix, the elements of which are all equal.

It has been shown in Chapter 3 that the double circuit line problem can be reduced to two decoupled single circuit lines problem and the line parameter matrices of these two equivalent single circuits are $([R_a]+[R_b])$, $([L_a]+[L_b])$, $([C_a]+[C_b])$ and $([R_a]-[R_b])$, $([L_a]-[L_b])$, $([C_a]-[C_b])$.

Denoting $[R_a]+[R_b]$ by $[R_s]$, $[L_a]+[L_b]$ by $[L_s]$, $[C_a]+[C_b]$ by $[C_s]$, $[R_a]-[R_b]$ by $[R_d]$, $[L_a]-[L_b]$ by $[L_d]$, $[C_a]-[C_b]$ by $[C_d]$, the diagonal elements of $[R_s]$, $[L_s]$, $[C_s]$ by R_{sd}, L_{sd}, C_{sd} respectively; the off-diagonal elements of $[R_s]$, $[L_s]$, $[C_s]$ by R_{sod}, L_{sod}, C_{sod} respectively; the diagonal elements of $[R_d]$, $[L_d]$, $[C_d]$ by R_{dd}, L_{dd}, C_{dd} respectively and the off-diagonal elements of $[R_d]$, $[L_d]$, $[C_d]$ by R_{dod}, L_{dod}, C_{dod} respectively, the matrix $[\Omega]$ in equations (E.11) and (E.13) (Appendix E) can be written for a transposed double circuit line as

$$[\Omega] = \begin{bmatrix} [\Omega_s] & [0] \\ [0] & [\Omega_d] \end{bmatrix}$$

where

$$[\Omega_s] = \begin{bmatrix} (L_{s0}/C_{s0})^{\frac{1}{2}} & 0 & 0 \\ 0 & (L_{s1}/C_{s1})^{\frac{1}{2}} & 0 \\ 0 & 0 & (L_{s1}/C_{s1})^{\frac{1}{2}} \end{bmatrix}$$

$$[\Omega_d] = \begin{bmatrix} (L_{d0}/C_{d0})^{\frac{1}{2}} & 0 & 0 \\ 0 & (L_{d1}/C_{d1})^{\frac{1}{2}} & 0 \\ 0 & 0 & (L_{d1}/C_{d1})^{\frac{1}{2}} \end{bmatrix}$$

$$[0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_{s0} = R_{sd} + 2R_{sod} \quad ; \quad R_{s1} = R_{sd} - R_{sod}$$

$$R_{d0} = R_{dd} + 2R_{dod} \quad ; \quad R_{d1} = R_{dd} - R_{dod}$$

$$L_{s0} = L_{sd} + 2L_{sod} \quad ; \quad L_{s1} = L_{sd} - L_{sod}$$

$$L_{d0} = L_{dd} + 2L_{dod} \quad ; \quad L_{d1} = L_{dd} - L_{dod}$$

$$C_{s0} = C_{sd} + 2C_{sod} \quad ; \quad C_{s1} = C_{sd} - C_{sod}$$

$$C_{d0} = C_{dd} + 2C_{dod} \quad \text{and} \quad C_{d1} = C_{dd} - C_{dod}$$

Here it can be observed that $R_{d1} = R_{s1}$; $L_{d1} = L_{s1}$; $C_{d1} = C_{s1}$ showing that there are only 3 distinct modes of propagation in the case of transposed double circuit lines.

6.3 FIRST CIRCUIT ENERGISATION TRANSIENT

The first step in the Uram and Miller's method is to express the vectors $\underline{k}_1$ and $\underline{b}$, in terms of the delayed functions $\underline{a}$ and $\underline{k}_2$ by forcing the boundary conditions, existing at the ends of the line.
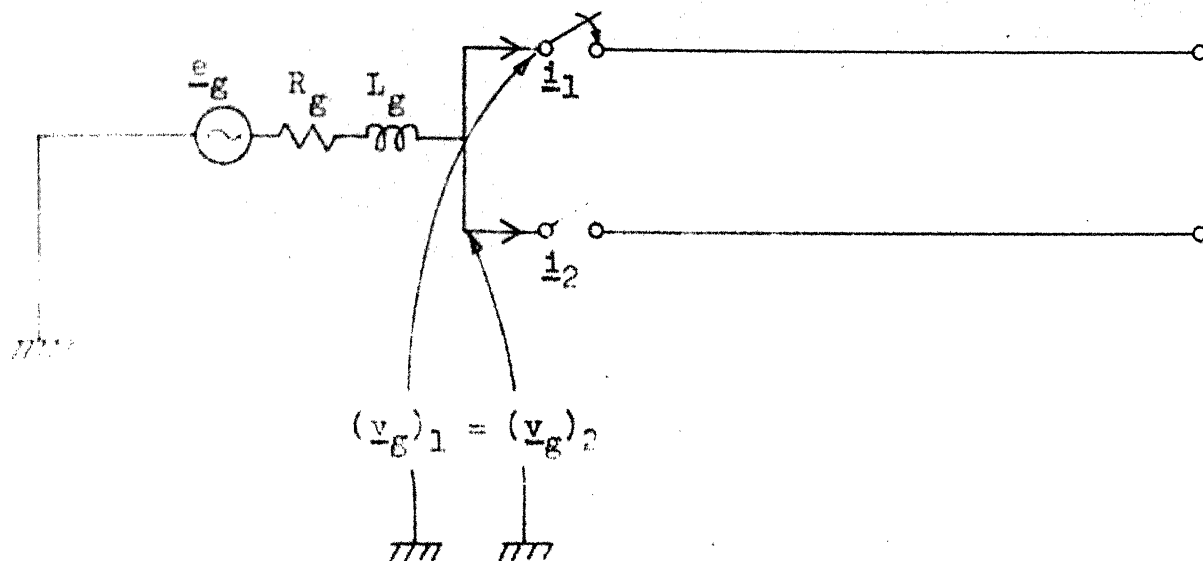
FIGURE 6.1: SINGLE LINE DIAGRAM - FIRST CIRCUIT
ENERGISATION - SECOND CIRCUIT OPEN.

Referring to Figure 6.1, which is a single line diagram for the first circuit energisation study of a double circuit line, open at the receiving end, the expression for the sending end current [Equation (E.11)] can be written as

$$\begin{bmatrix} \underline{i}_1 \\ \underline{i}_2 \end{bmatrix} = \begin{bmatrix} [Q] & -[Q] \\ [Q] & [Q] \end{bmatrix} \begin{bmatrix} [\Omega_s]^{-1} & [0] \\ [0] & [\Omega_d]^{-1} \end{bmatrix} \begin{bmatrix} (\underline{k}_1)_1 & -(\underline{k}_2)_1 \\ (\underline{k}_1)_2 & -(\underline{k}_2)_2 \end{bmatrix}$$

$$(6.1)$$

where $\underline{k}_1$ and $\underline{k}_2$ are partitioned as

$$\underline{k}_1 = \begin{bmatrix} (\underline{k}_1)_1 \\ (\underline{k}_1)_2 \end{bmatrix} \quad \text{and} \quad \underline{k}_2 = \begin{bmatrix} (\underline{k}_2)_1 \\ (\underline{k}_2)_2 \end{bmatrix}$$

The expression for the sending end voltage [equation (E.10)] can be written as

$$\begin{bmatrix} (\underline{v}_g)_1 \\ (\underline{v}_g)_2 \end{bmatrix} = \begin{bmatrix} [Q] & -[Q] \\ [Q] & [Q] \end{bmatrix} \begin{bmatrix} (\underline{k}_1)_1 + (\underline{k}_2)_1 \\ (\underline{k}_1)_2 + (\underline{k}_2)_2 \end{bmatrix} \qquad (6.2)$$

As the second circuit is unenergised, forcing the condition $\underline{i}_2 = \underline{0}$ in equation (6.1) yields

$$[\Omega_s]^{-1} \{ (\underline{k}_1)_1 - (\underline{k}_2)_1 \} + [\Omega_d]^{-1} \{ (\underline{k}_1)_2 - (\underline{k}_2)_2 \} = \underline{0}$$

i.e. $\quad (\underline{k}_1)_2 = (\underline{k}_2)_2 + [\Omega_d][\Omega_s]^{-1} \{ (\underline{k}_2)_1 - (\underline{k}_1)_1 \}$ (6.3)

The other sending end terminal condition is determined by the source impedance. This can be written as

$$\underline{E}_g - R_g[Q]\{[\Omega_s]^{-1}[(\underline{k}_1)_1 - (\underline{k}_2)_1] - [\Omega_d]^{-1}[(\underline{k}_1)_2 - (\underline{k}_2)_2]\}$$

$$-L_g[Q]\{[\Omega_s]^{-1}[\frac{d(\underline{k}_1)_1}{dt} - \frac{d(\underline{k}_2)_1}{dt}] - [\Omega_d]^{-1}[\frac{d(\underline{k}_1)_2}{dt} - \frac{d(\underline{k}_2)_2}{dt}]\}$$

$$= [Q]\{(\underline{k}_1)_1 + (\underline{k}_2)_1 - (\underline{k}_1)_2 - (\underline{k}_2)_2\}$$

Expressing $(\underline{k}_1)_2$ in terms of $(\underline{k}_1)_1$ as per equation (6.3), the above equation can be manipulated to the form

$$\frac{d(\underline{k}_1)_1}{dt} = \frac{1}{2L_g}[\Omega_s][Q]^{-1}\underline{E}_g - [\frac{R_g}{L_g} + \frac{1}{2L_g}\{[\Omega_d] + [\Omega_s]\}](\underline{k}_1)_1$$

$$+ [\frac{R_g}{L_g} + \frac{1}{2L_g}\{[\Omega_d] - [\Omega_s]\}](\underline{k}_2)_1 + \frac{1}{L_g}[\Omega_s](\underline{k}_2)_2 + \frac{d(\underline{k}_2)_1}{dt}$$

$$(6.4)$$

By forcing the condition that the line is open at the receiving end, we get the relationship,

$$\underline{b} = \underline{a} \qquad\qquad (6.5)$$

The delay functions $\underline{a}$ and $\underline{k}_2$ are related to the functions $\underline{k}_1$ and $\underline{b}$ as follows:

$$a_1 = e^{-(R_{s0}/2)\sqrt{(C_{s0}/L_{s0})}\ell} \ [k_{11}(t - \tau_{s0})]_1$$

$$a_2 = e^{-(R_{s1}/2)\sqrt{(C_{s1}/L_{s1})}\ell} \ [k_{12}(t - \tau_{s1})]_1$$

$$a_3 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \cdot [\underline{k}_{13}(t - \tau_{s1})]_1$$

$$a_4 = e^{-(R_{d0}/2)\,\sqrt{(C_{d0}/L_{d0})}\,\ell} \, [k_{11}(t - \tau_{d0})]_2$$

$$a_5 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \, [k_{12}(t - \tau_{s1})]_2$$

$$a_6 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \, [k_{13}(t - \tau_{s1})]_2$$

$$(6.6)$$

where $\tau_{s0} = \ell\,\sqrt{(L_{s0}\,C_{s0})}$; $\tau_{s1} = \ell\,\sqrt{(L_{s1}\,C_{s1})}$ and

$$\tau_{d0} = \ell\,\sqrt{(L_{d0}\,C_{d0})}.$$

$$(k_{21})_1 = e^{-(R_{s0}/2)\,\sqrt{(C_{s0}/L_{s0})}\,\ell} \, b_1(t - \tau_{s0})$$

$$(k_{22})_1 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \, b_2(t - \tau_{s1})$$

$$(k_{23})_1 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \, b_3(t - \tau_{s1})$$

$$(6.7)$$

$$(k_{21})_2 = e^{-(R_{d0}/2)\,\sqrt{(C_{d0}/L_{d0})}\,\ell} \, b_4(t - \tau_{d0})$$

$$(k_{22})_2 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \, b_5(t - \tau_{s1})$$

$$(k_{23})_2 = e^{-(R_{s1}/2)\,\sqrt{(C_{s1}/L_{s1})}\,\ell} \, b_6(t - \tau_{s1})$$

With the help of equations $(6.3)-(6.7)$ an incremental solution for the four vectors $\underline{k}_1$, $\underline{k}_2$, $\underline{a}$ and $\underline{b}$

at each instant of time and hence the response at any point of interest on the transmission line can be evaluated. As equation (6.4) is a differential equation, a numerical solution of this equation becomes necessary to arrive at the values of $(\underline{k}_1)_1$ at each instant.

6.4 SECOND CIRCUIT ENERGISATION

An algorithm for studying the transient due to the energisation of the second circuit, when the first circuit is in steady state is developed in this Section. The voltage cancellation technique and the principle of superposition are used for simulating the circuit breaker closing operation. The principle of superposition is applicable for this part of the analysis as the system considered is linear.

The steps involved can be described with the help of Figures 6.2.1 and 6.2.2 as follows:

(i)   First assuming that the second circuit continues to be unenergised, the response at any point of interest on the line (particularly the voltage at the open receiving end) and the voltages across the circuit breaker terminals at the sending end of the second circuit are calculated. As the first circuit is in steady state, this step of the calculation is a steady state analysis.

(ii)   The voltages across the breaker poles, calculated in step (i) are injected across the poles with reversed polarity as cancellation voltages and the sending end

FIGURE 6.2.1

$v_{g3} - v_{\ell3} = v_{13}$

$v_{g2} - v_{\ell2} = v_{12}$
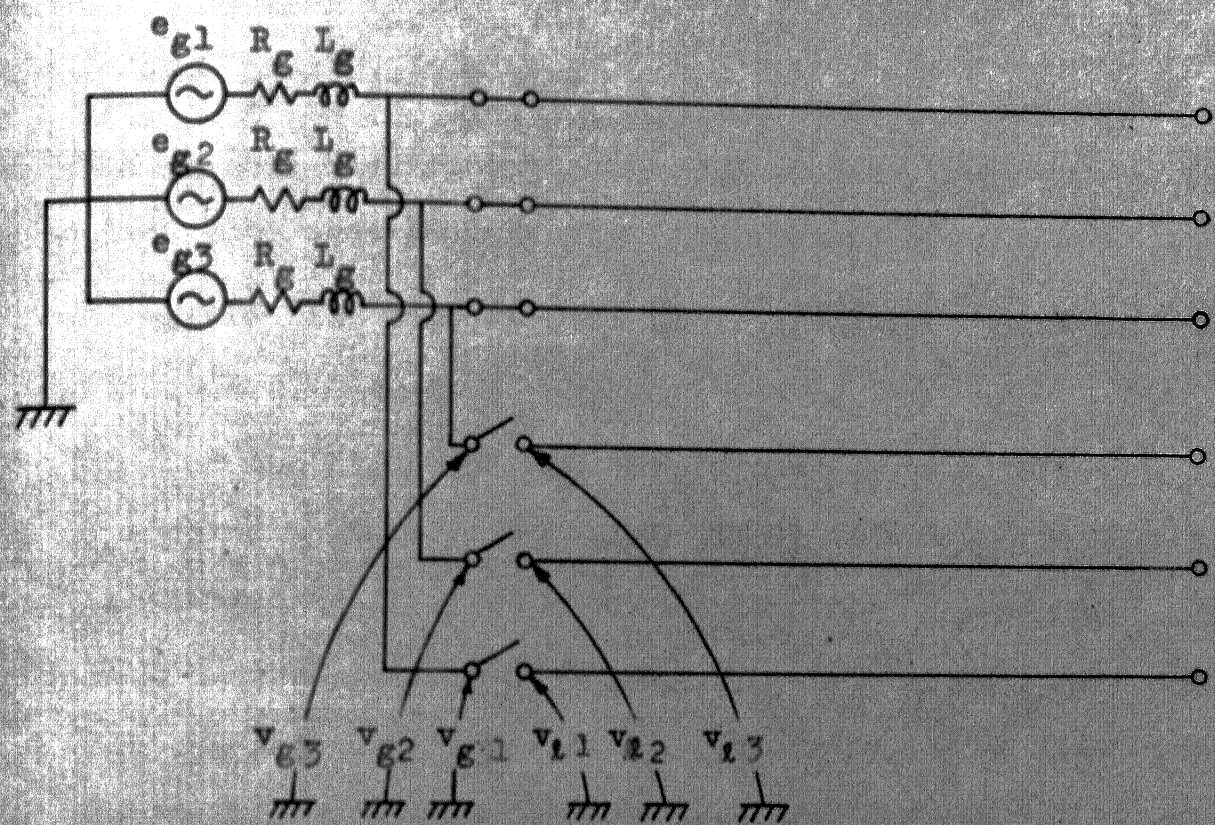
$v_{g1} - v_{\ell1} = v_{11}$
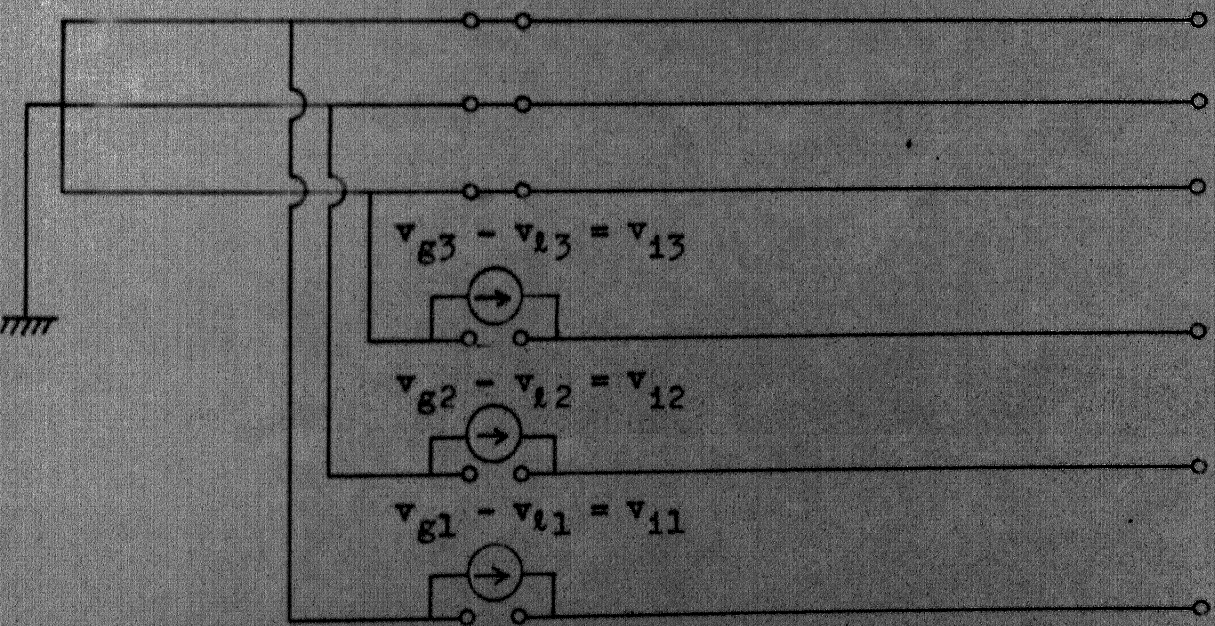
FIGURE 6.2.2

PRINCIPLE OF SUPERPOSITION USED FOR SIMULATION OF
SECOND CIRCUIT ENERGISATION - FIRST CIRCUIT IN
STEADY STATE.

terminals of the first circuit are grounded. Now with these cancellation voltages alone present, the response at the above mentioned point of interest is again calculated.

(iii) Superimposing these two component responses, the total response is obtained.

The algorithm for finding the response to the injected voltage (step ii) by Uram and Miller's method is developed as follows:

Referring to Figure 6.2.2, the terminal conditions at the sending end are that the voltages of the first circuit are zero and those of the second circuit are v_{i1}, v_{12} and v_{13}.

Forcing these conditions and using the same notation, as used for the first circuit energisation case, equation (6.2) becomes

$$\begin{bmatrix} [Q] & -[Q] \\ [Q] & [Q] \end{bmatrix} \begin{bmatrix} (\underline{k}_1)_1 + (\underline{k}_2)_1 \\ (\underline{k}_1)_2 + (\underline{k}_2)_2 \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \underline{v}_i \end{bmatrix} \qquad (6.8)$$

where $\quad \underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad$ and $\quad \underline{v}_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \end{bmatrix}$

i.e. $\quad [Q] \{ (\underline{k}_1)_1 + (\underline{k}_2)_1 + (\underline{k}_1)_2 + (\underline{k}_2)_2 \} = \underline{v}_i \qquad (6.9)$

and $\quad (\underline{k}_1)_1 = (\underline{k}_1)_2 - (\underline{k}_2)_1 + (\underline{k}_2)_2 \qquad (6.10)$

Substituting for $(\underline{k}_1)_1$ in equation (6.9) by equation (6.10), we get

$$(\underline{k}_1)_2 = \tfrac{1}{2}[Q]^{-1} \underline{v}_i - (\underline{k}_2)_2 \qquad (6.11)$$

Substituting (6.11) in (6.10), we get

$$(\underline{k}_1)_1 = \tfrac{1}{2}[Q]^{-1} \underline{v}_i - (\underline{k}_2)_1 \qquad (6.12)$$

As the receiving end is open, from equation (E.13), we get

$$\underline{b} = \underline{a} \qquad (6.13)$$

The functions $(\underline{k}_1)_2$, $(\underline{k}_1)_1$ and $\underline{b}$ have been expressed in terms of delayed functions and known quantities $[Q]$, $\underline{v}_i$ by equations (6.11) - (6.13) respectively.

The delay functions $\underline{a}$ and $\underline{k}_2$ are related to the functions $\underline{k}_1$ and $\underline{b}$ by the sets of equations (6.6) and (6.7). Hence with the help of equations (6.6), (6.7), (6.11), (6.12) and (6.13), an incremental solution for the desired response for the injected voltage can be calculated.

6.5 COMPUTATIONAL RESULTS

The same example, tried in Chapter 5, using the modified Fourier transform method is taken up and the Uram and Miller's method is used for evaluating both the first and the second circuit energising transients on the double circuit line. The modal components of the parameters for this line are given below.

$R_{s0} = 0.321396 \ \Omega/mile$; $L_{s0} = 0.00686016$ H/mile ;

$C_{s0} = 0.934644 \times 10^{-8}$ f/mile

$R_{s1} = R_{d1} = 0.030330 \ \Omega/mile$; $L_{s1} = L_{d1} = 0.00142587$ H/mile

$C_{s1} = C_{d1} = 2.085753 \times 10^{-8}$ f/mile ; $R_{d0} = 0.03 \ \Omega/mile$

$L_{d0} = 0.00187236$ H/mile ; $C_{d0} = 1.620324 \times 10^{-8}$ f/mile.

The phase voltage waveforms at the receiving end of the first circuit, subsequent to the closing of the first circuit are plotted in Figure 6.3. The breakers of all the three phases are assumed to close simultaneously at the instant when the source voltage of phase 'a' is at positive peak. A maximum peak of 2.32 p.u. is observed to occur on phase 'a' at 9.2 m.secs. for this case. The corresponding voltage waveforms, obtained using the modified Fourier transform method are also plotted in the same figure. It can be observed that there is a close matching between the waveforms obtained by both the methods.

Next for the same line, the transient due to the closing of the second circuit, when the first circuit is in steady state (described in Section 6.4) is studied. The steady state analysis of Section 5.5.2 holds good here also and hence in step (ii) of Section 6.4, it can be assumed that no voltages are induced on the second circuit conductors. Hence, the voltages across the breaker poles of the second circuit (which is the injected voltage source) can be taken

TIME, MS

×——×——× URAM AND MILLER'S METHOD

⊙--●--● MODIFIED FOURIER TRANSFORM METHOD
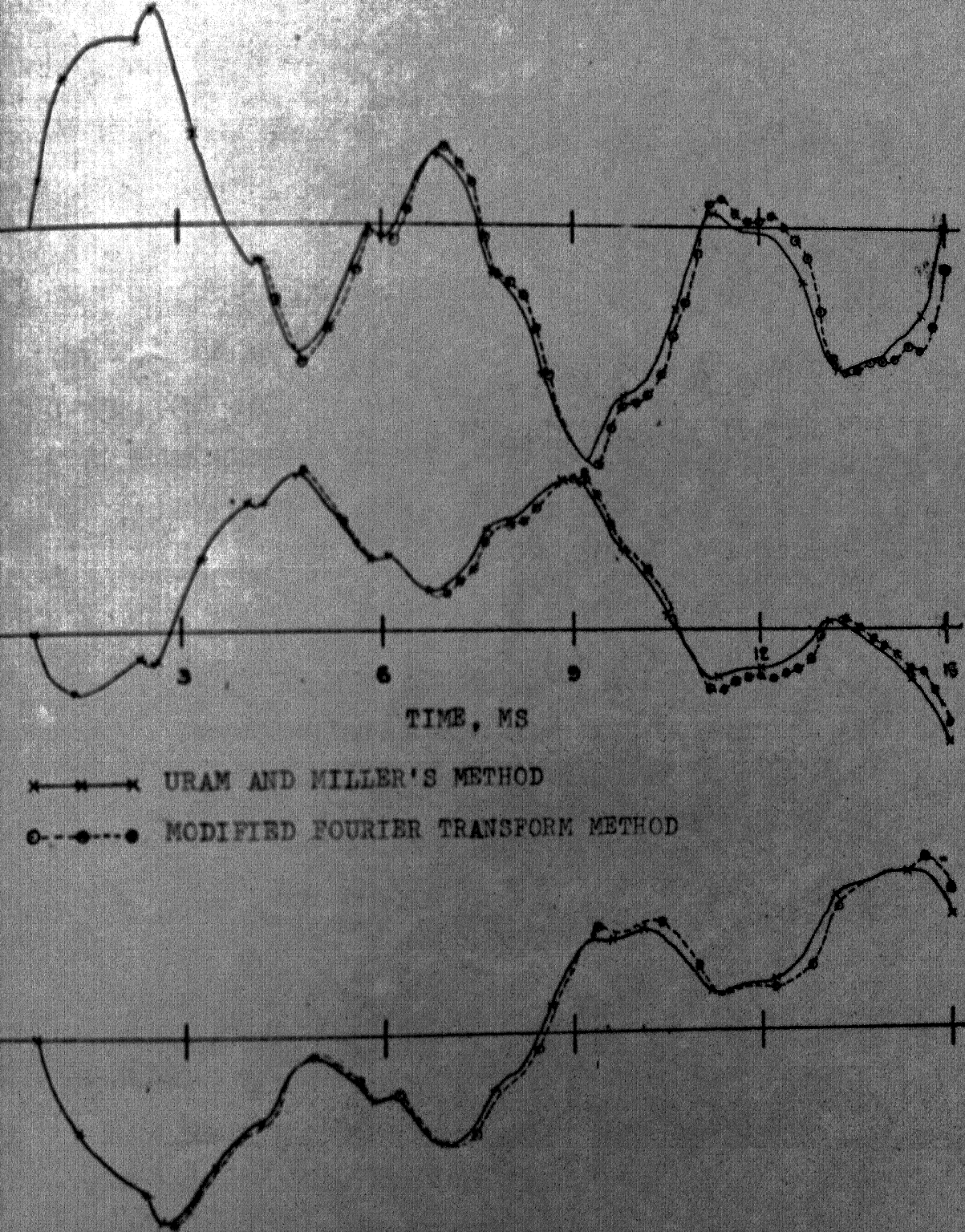
FIGURE 6.3: COMPARISON OF VOLTAGE WAVEFORMS AT RECEIVING END
BY URAN AND MILLER AND MODIFIED FOURIER TRANSFORM
METHODS – FIRST CIRCUIT ENERGISATION – SECOND
CIRCUIT OPEN.
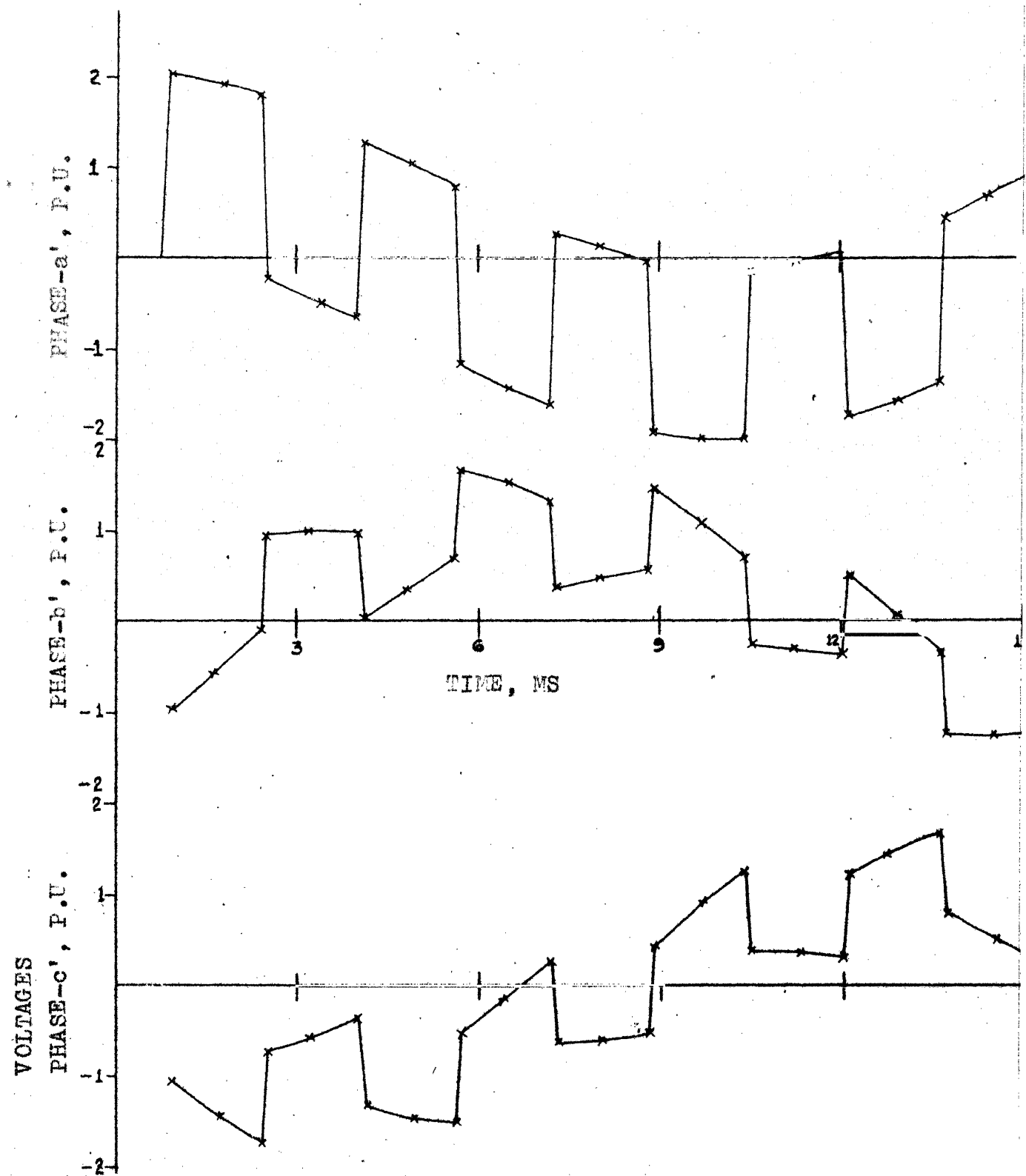
FIGURE 6.4: VOLTAGE WAVEFORMS AT RECEI
ENERGISATION — FIRST CIRCU

as the steady state voltages at the sending end of the
connected phases of the first circuit. Again the breaker
poles of all the three phases of the second circuit are
assumed to close · simultaneously at the instant, when the
phase-a' voltage is at positive peak. The phase voltage
waveforms at the receiving end of the second circuit,
subsequent to the closing of the second circuit are
plotted in Figure 6.4. A maximum peak of 2.02 p.u. is
observed to occur for this case at 0.9 m.secs. on phase-a'.
These waveforms are almost coinciding with the corresponding
ones, obtained using the modified Fourier transform method
in Chapter 5.

6.6 CONCLUSIONS

The extension of the theory of Uram and Miller's
method for solving the energisation transients on a trans-
posed double circuit line has been presented in this chapter
The same line, tried in Chapter 5 using the modified Fourier
transform method, has been taken up and the transient due to
the energisation of the first circuit alone, with the second
circuit remaining unenergised and the transient due to the
energisation of the second circuit, when the first circuit
is operating in steady state, have been studied. The voltage
waveforms at the open receiving end of the line, obtained
using both the Uram and Miller's and the modified Fourier
transform methods have been compared. The results by both

the methods have been found to be closely matching. As
the Uram and Miller's method is a time domain method, the
frequency dependence of line parameters cannot be directly
incorporated in this method. The untransposed lines also
cannot be handled by this method. However, this method
takes relatively less computer time.

ENERGISATION TRANSIENT DUE TO SEQUENTIAL CLOSING OF CIRCUIT BREAKER   POLES

7.1 INTRODUCTION

In reality, during the energisation of a transmissio[n] line, the poles of the circuit breaker do not close simulta[neo]ously. However, such an assumption is generally made only [to] simplify the analysis and to get an idea of the magnitude [of] the switching overvoltages involved. The difference in ti[me] in the closing of different poles of a circuit breaker is small, but this time is comparable to the time period of th[e] electrical transients in the overvoltages that develop. Th[is] has been seen in earlier chapters to be of the order of a f[ew] m.secs. Therefore, for an accurate analysis, the non-simultaneous or sequential closing of the circuit breaker poles has to be incorporated in the program. The sequentia[l] closing of the breaker poles is bound to give rise to a hig[h] order of overvoltages. There are basically two approaches [in] the existing literature for the incorporation of sequential closing feature in the Fourier transform method of studying the energisation transient on a transmission line. Wedepohl and Mohamed [27] have proposed a method of solving the prob[lem] by numerically evaluating the Fourier transform of the indu[ced] voltages on the unenergised phases, known at discrete intervals of time. On the other hand, the method proposed

by Battison et al [28] solves the problem by using a
convolution integral approach for taking into account the
sequential pole closure. In this chapter, the method
proposed by Wedepohl and Mohamed is extended for the case
of a double circuit line. After presenting the relevant
theory for this extension, the double circuit line example,
with receiving end open, of Chapters 5 and 6 is considered.
The transient, arising due to the sequential closure of the
circuit breaker poles of the second circuit, when the first
circuit is operating in steady state, is evaluated. The
transient voltage waveforms obtained by the double circuit
modelling is compared with those obtained by ignoring the
presence of the first circuit. Also the effect of the
continuous variation of line parameter values with frequency
on the energisation transient for both transposed and un-
transposed lines is investigated.

7.2 INCORPORATION OF SEQUENTIAL CLOSING OF CIRCUIT BREAKER POLES IN FOURIER TRANSFORM METHOD

The sequential closing of the circuit breaker poles
involves changes in the terminal conditions at the sending
end of a transmission line at the closing instant of each
breaker pole. As the Fourier transform method is a frequency
domain method and because the change in system conditions
due to sequential closing occurs at different instants of
time, an indirect solution of the problem becomes essential.

To start with, when only the first phase of the line is getting closed, the conditions to be forced at the sending end are that the currents on the other two unenergised phases are zero. Further, the voltage and current on the first phase at the sending end are determined by the source voltage and the impedance, present on this phase at this end. After a short time of the order of a few m.secs., the second pole gets closed, leaving only the third phase unenergised. At this stage, the current through the third phase is zero and the voltages and currents in the first and second phases at the sending end are determined by the source voltages and impedances present on these phases at the sending end. After the elapse of a few more m.secs., the third pole gets closed and from this instant onwards, the voltages and currents on all the three phases are determined by the source voltages and impedances at the sending end. During the interval between the first and second pole closure, voltages are induced on the floating second and third phase conductors. In the Fourier transform method, these induced voltages at the sending end should be first calculated as functions of time and then the modified Fourier transform of these functions so calculated at discrete interval of time should be numerically evaluated to be fed as input in frequency domain. Similarly during the interval between the second and third pole closure, voltage is induced on the floating third phase conductor because of the mutual effect between

this phase and the other two energised phases. The modified Fourier transform of this function in this time interval, augmented by the induced voltage on this phase during the interval between the first and second pole closure is to be numerically computed and fed as input voltage in frequency domain to this phase.

The steps involved in the solution of the sequential closing transient can be summarized as follows:

(i) The first pole closes (say at zero time) and the sending end boundary conditions at this stage are a known voltage on the first phase and zero currents on the other two phase conductors. Using the modified Fourier transform method, the system response at the point of interest is computed as a function of time. In particular, the induced voltages at the sending end on the two unenergised phase conductors are also computed.

(ii) After a time interval T_1, the second pole closes and the new boundary conditions at the sending end are known voltages on the first two phases and zero current on the third phase. The transformed voltages to be fed as input on the second phase includes the modified Fourier transform of the induced voltage on this conductor in the interval $0 <$ time $< T_1$ (evaluated numerically) in addition to the transform of the source voltage, connected to this phase for time $> T_1$ (for which an analytical expression can be obtained). For these terminal conditions, the response

at the point of interest can be computed as a function
of time. In particular, the induced voltage on the floating
third phase conductor is also to be computed. These res-
ponses are valid right from zero time. But the response,
so obtained will be a mere repetition of the corresponding
response, obtained in the first step in the interval
$0 < t < T_1$. However, if it is computed again here, it
can be utilized to get an idea of the error, introduced
by the numerical integration and the numerical evaluation
of the transform of the induced voltages involved in this
step.

(iii) After an elapse of time, finally the third pole
closes at time T_2. Now the transformed voltages on all the
three phases at the sending end are known and hence the
response at the point of interest for time $> T_2$ can be
computed. Here the transformed voltages, to be fed at the
sending end as input, includes in addition to the analytically
known transformed voltages on all the three phases, the
numerically evaluated transforms of the induced voltage on
second phase in the interval $0 < time < T_1$ and that of the
induced voltage on the third phase in the interval
$0 < time < T_2$. Again the computing of the response right
from zero instant helps in giving an idea of the error,
introduced by the numerical processes involved.

7.3 DEVELOPMENT OF ALGORITHM FOR HANDLING SEQUENTIAL CLOSING

As described in the last section, the incorporation of sequential closing essentially involves finding the transient response of the line for **three different** sending end terminal conditions. The change in terminal conditions occurs at the closing instant of each pole. The terminal conditions are zero currents on unenergised phases and known voltages on the energised phases behind source reactances, which can be converted to equivalent current sources with appropriate shunt admittances. Hence, once a set of equations are formed for computing the transient response of the line for known sending end currents, it can be used repeatedly thrice with the suitable currents injected at each of the pole closing instants, to get the complete response.

It has been derived in Section 5.3 that when a transmission line is viewed as a two port network, the transformed currents and voltages at the terminals are related by the expression,

$$
\begin{bmatrix} \underline{I}_s \\ \underline{I}_R \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [A] \end{bmatrix} \begin{bmatrix} \underline{V}_s \\ \underline{V}_R \end{bmatrix} \tag{7.1}
$$

where $[A] = [Z]^{-1}[\psi][\text{Coth}\,\psi\ell]$

$[B] = -[Z]^{-1}[\psi][\text{Cosech}\,\psi\ell]$

$[Z] = [R]+(a+j\omega)[L]$

$$[Y] = (a+j\omega)[C]$$

$$[\psi] = [Q][\gamma][Q]^{-1}$$

$$[\text{Coth } \psi \ell] = [Q][\text{Coth } \gamma \ell][Q]^{-1}$$

$$[\text{Cosech} \psi \ell] = [Q][\text{Cosech } \gamma \ell][Q]^{-1}$$

and $[\gamma]$ is the diagonal matrix, whose diagonal elements
are square roots of the eigen values of the
matrix product $[Z][Y]$.

When the open receiving end condition is forced into
equation (7.1), V_R and V_S can be expressed in terms of
I_S as follows:

$$V_S = ([A] - [B][A]^{-1}[B])^{-1} I_S \tag{7.2}$$

$$V_R = ([B] - [A][B]^{-1}[A])^{-1} I_S \tag{7.3}$$

When there is a shunt admittance $[Y_S]$ at the sending
end, equations (7.2) and (7.3) take the form

$$V_S = \{ [A]+[Y_S]-[B][A]^{-1}[B]\}^{-1} I_S \tag{7.4}$$

$$V_R = \{ [B]-([A]+[Y_S])[B]^{-1}[A]\}^{-1} I_S \tag{7.5}$$

where now I_S includes the current drawn by the shunt
admittance in addition to the current drawn by the line.

In the equations (7.4) and (7.5), the quantities

$[Y_S]$ and I_S vary according to whether the different phase
poles of the breaker at the sending end is open or closed.

Initially, in the interval $0 < t < T_1$, when only the first pole is closed, $[Y_s]$ and $\underline{I}_s$ are given by the expressions

$$[Y_s] = \begin{bmatrix} 1/[R_s+(a+j\omega)L_s] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{I}_s = \begin{bmatrix} E_{s1}/[R_s+(a+j\omega)L_s] \\ 0 \\ 0 \end{bmatrix}$$

where E_{s1} is the modified Fourier transform of the first phase source voltage and R_s, L_s are the source resistance and inductance per phase respectively.

In the interval $T_1 < t < T_2$, the first and second poles are closed and now

$$[Y_s] = \begin{bmatrix} 1/[R_s+(a+j\omega)L_s] & 0 & 0 \\ 0 & 1/[R_s+(a+j\omega)L_s & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{I}_s = \begin{bmatrix} E_{s1}/[R_s+(a+j\omega)L_s] \\ (E_{s2}+V_{s2})/[R_s+(a+j\omega)L_s] \\ 0 \end{bmatrix}$$

where E_{s2} is the modified Fourier transform of the source voltage of this phase (which becomes the line side voltage also on this phase from time T_1 onwards) [i.e. the modified Fourier transform of $e_{s2}(t-T_1)\ u\ (t-T_1)$] and V_{s2} is the

numerically evaluated modified Fourier transform of the induced voltage on the second phase conductor at the sending end in the interval $0 < t < T_1$.

From the instant T_2 onwards, all the three poles are closed and hence

$$[Y_S] = \begin{bmatrix} 1/[R_S+(a+j\omega)L_S] & 0 & 0 \\ 0 & 1/[R_S+(a+j\omega)L_S] & 0 \\ 0 & 0 & 1/[R_S+(a+j\omega)L_S] \end{bmatrix}$$

$$\underline{I}_S = \begin{bmatrix} E_{s1}/[R_S+(a+j\omega)L_S] \\ (E_{s2}+V_{s2})/[R_S+(a+j\omega)L_S] \\ (E_{s3}+V_{s3})/[R_S+(a+j\omega)L_S] \end{bmatrix}$$

where E_{s3} is the modified Fourier transform of the source voltage of this phase (which becomes the line side voltage on this phase from time T_2 onwards) [i.e. the modified Fourier transform of $e_{s3}(t - T_2)$ u $(t - T_2)$] and V_{s3} is the numerically evaluated modified Fourier transform of the induced voltage on the third phase conductor at the sending end in the interval $0 < t < T_2$.

The appropriate values of $[Y_S]$ and $\underline{I}_S$, when used in expressions (7.4) and (7.5) in the appropriate intervals of time, facilitates calculation of the desired transient overvoltage response in frequency domain and it can be converted to time domain by computing numerically the inverse transformation integral.

Here it can be observed that the numerical integration of the transformed response to time domain can be limited to the extent of calculating the induced voltages at the sending end on the second and third phase floating conductors in the interval $0 < t < T_1$, subsequent to the first pole closing and calculating the induced voltage at the sending end on the third phase conductor alone in the interval $T_1 < t < T_2$, subsequent to second pole closing. The transforming back of the receiving end voltages to time domain may be done just once at the end.

The method of calculating numerically the modified Fourier transform of the induced voltages on unenergised conductors, known at discrete interval of time is given in the following section.

A flow chart of the program for computing the energisation transient on a single circuit tranmission line with sequential closing of circuit breaker poles by modified Fourier transform method is presented in Appendix F.

7.4 NUMERICAL EVALUATION OF THE MODIFIED FOURIER TRANSFORM

For numerically evaluating the modified Fourier transform of a known function f(t), it can be divided into N strips of equal width, the interval of the strip being sufficiently small, such that the variation of the function within any of these intervals can be approximated by a

straight line. Then the modified Fourier transform of the function $f(t)$ can be obtained by summing up the modified Fourier transforms of all these individual strips. A typical strip of the function and how it can be split up into components of step and ramp functions are shown in Figures 7.1.1 and 7.1.2. This strip can be split into two component step functions (one of magnitude f_{n-1}, starting at instant $(n-1) \Delta t$, the other of magnitude $-f_n$, starting after a delay Δt at the instant $n \Delta t$) and two ramp components (one of slope $\dfrac{\Delta f_n}{\Delta t}$, starting at instant $(n-1) \Delta t$ and the other of slope $-\dfrac{\Delta f_n}{\Delta t}$, starting after a delay Δt at the instant $n \Delta t$).

When the modified Fourier transforms of the two component steps of all such strips are summed up, the transform of all such steps in the entire range of time is obtained as

$$F_s(\omega) = \frac{1}{(a+j\omega)} [f_o - f_N e^{-(a+j\omega)N\Delta t}] \qquad (7.6)$$

The sum of the modified Fourier transforms of all component ramps can be given by

$$F_r(\omega) = - \frac{1}{\Delta t(a+j\omega)^2} \sum_{n=1}^{N} \Delta f_n [e^{-(a+j\omega)(n-1)\Delta t} - e^{-(a+j\omega)n\Delta t}] \qquad (7.7)$$

An economy in the computation of $F_r(\omega)$ can be effected by expanding and grouping the consecutive terms
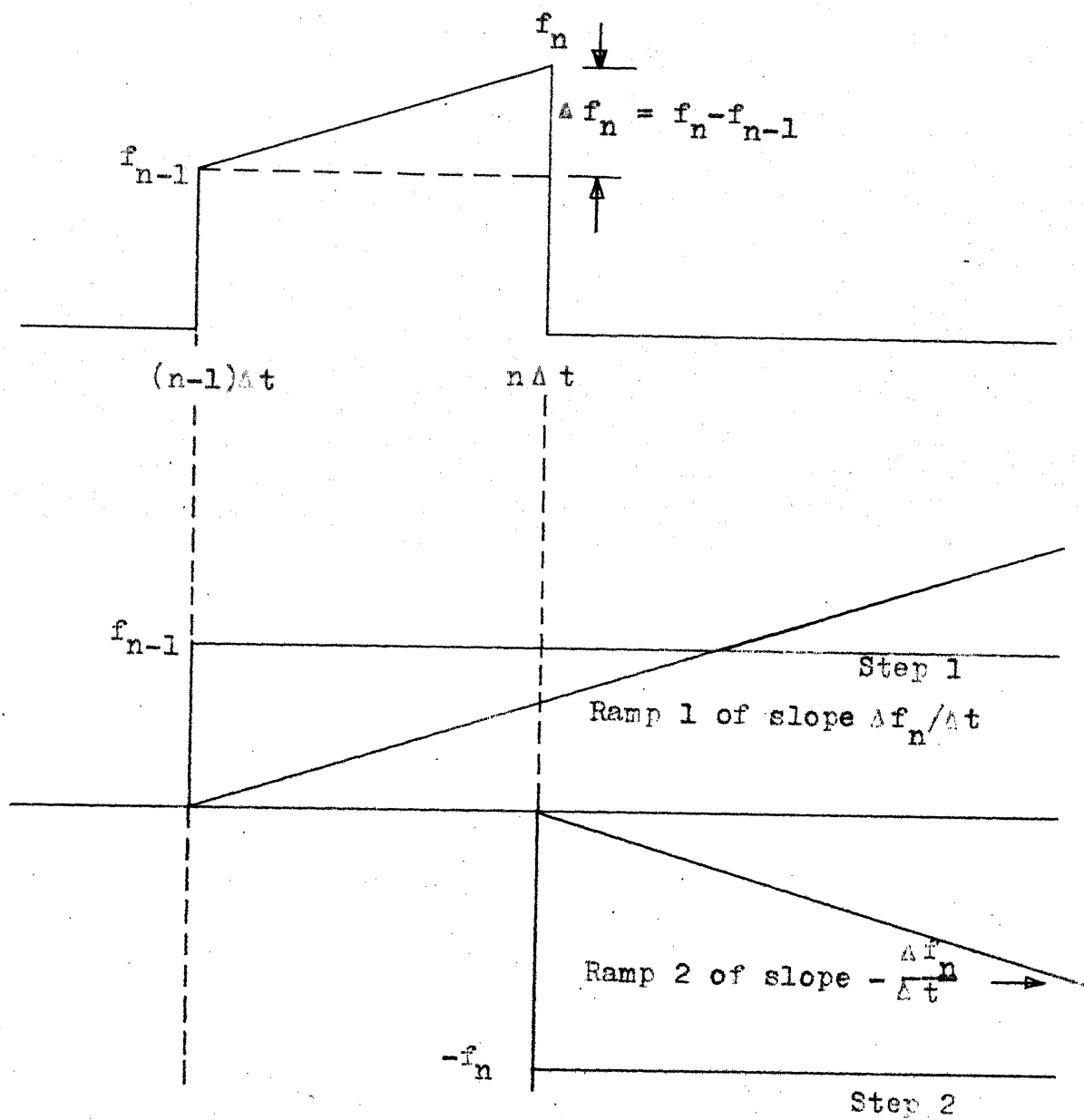
FIGURE 7.1: NUMERICAL EVALUATION OF FOURIER TRANSFORM
OF A FUNCTION.

of (7.7) together. By this manipulation, equation (7.7) can be reduced to the form

$$F_r(\omega) = \frac{1}{\Delta t(a+j\omega)^2} [f_1 - f_0 - (f_N - f_{N-1})\, e^{-(a+j\omega)N\Delta t}$$

$$+ \sum_{n=1}^{N-1} (f_{n+1} - 2f_n + f_{n-1})\, e^{-(a+j\omega)n\Delta t}]$$

Hence the modified Fourier transform of the function f(t) in the whole range of time can be written as

$$F(\omega) = \frac{1}{(a+j\omega)} [f_0 \{1 - \frac{1}{\Delta t(a+j\omega)}\} + \frac{f_1}{\Delta t(a+j\omega)}$$

$$+ \{\frac{f_{N-1}}{\Delta t(a+j\omega)} - f_N(1 + \frac{1}{\Delta t(a+j\omega)})\}\, e^{-(a+j\omega)N\Delta t}$$

$$+ \frac{1}{\Delta t(a+j\omega)} \sum_{n=1}^{N-1} (f_{n+1} - 2f_n + f_{n-1})\, e^{-(a+j\omega)n\Delta t}]$$

7.5 SINGLE CIRCUIT LINE EXAMPLES

7.5.1 Verification of Computer Program:

To check the computer program, developed using the modified Fourier transform method, incorporating the sequential closing of circuit breaker poles, the transient voltage at the receiving end of a typical 400 KV single circuit line, computed by this method is compared with the results, obtained for the same example by Uram and Miller's method.

This transmission line of Uttar Pradesh State Electricity Board is 400 kms. long and connects Lucknow with Obra. It has got two shunt reactors, one each at sending and receiving ends. The energisation study is done for energisation at Obra end. Figure 7.2 gives the system set up. The system data are given in Appendix G.

The phase sequence of closing of the circuit breaker poles is assumed for the study to be RBY and the switching instants of B phase and Y phase are taken as 3.3 m.secs. and 6.7 m.secs. respectively (taking the instant of closing of R phase as zero time). This set of switching instants are such that each pole gets closed when the source voltage of that phase is at peak value.

The phase voltage waveforms at the receiving end, obtained using both the methods are compared in Figure 7.3. There is a close matching between the waveforms, obtained by the two methods. A maximum peak of -3.01 p.u. in the case of the modified Fourier transform method and -2.88 p.u. in the case of the Uram and Miller's method are observed to occur, in both the cases on phase R at 9 m.secs.

This particular example contains reactors at both the ends of the line additionally as compared to the system, considered in Section 7.3, while developing the necessary theory for incorporating sequential closing feature in the modified Fourier transform method. When
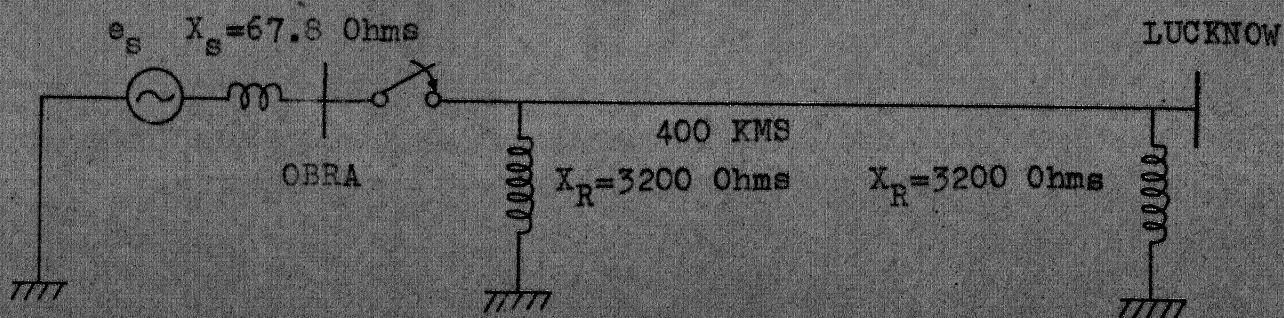
FIGURE 7.2: SYSTEM DIAGRAM OF THE EXAMPLE
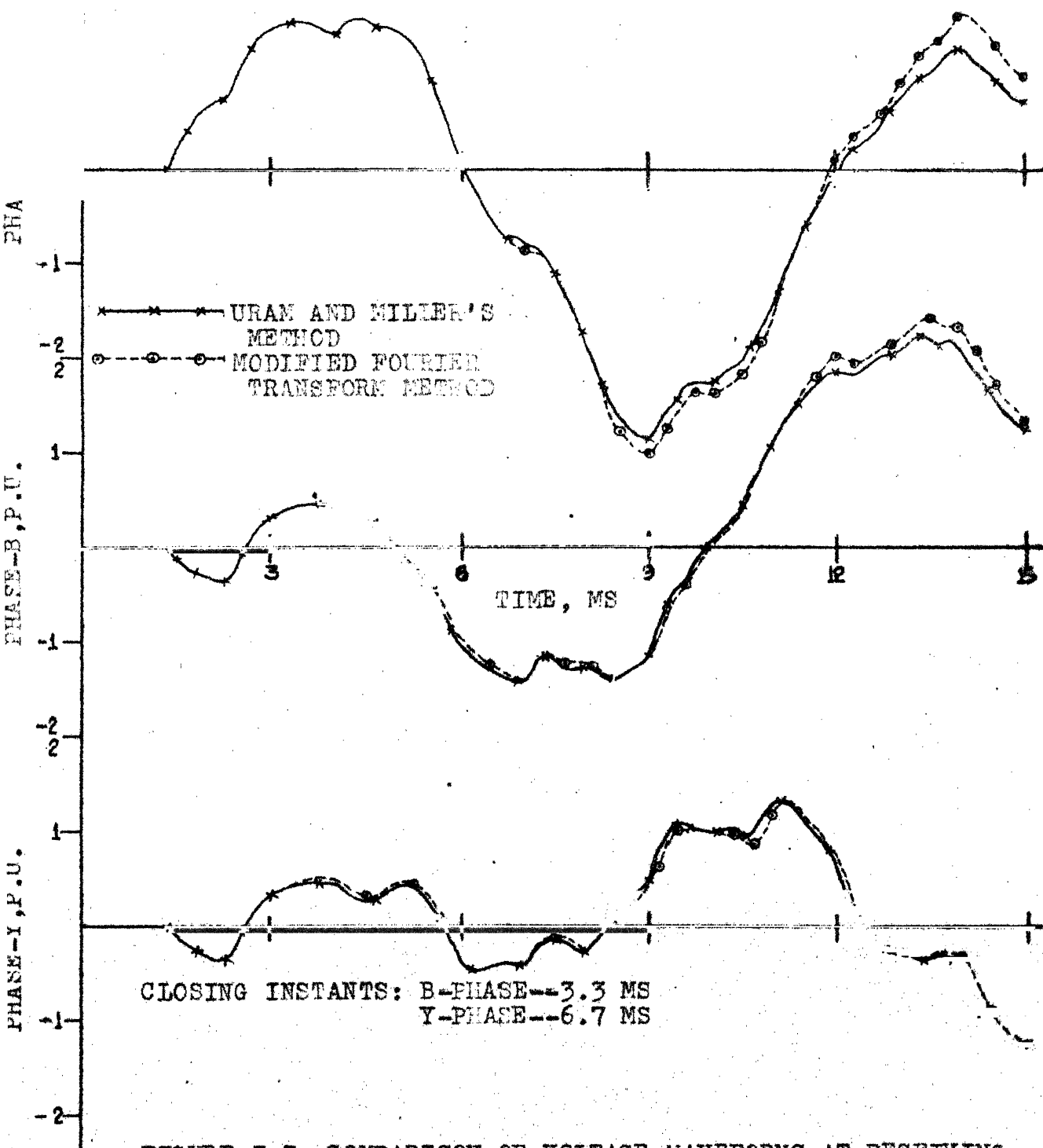
FIGURE 7.3: COMPARISON OF VOLTAGE WAVEFORMS AT RECEIVING
END BY URAM AND MILLER AND MODIFIED FOURIER
TRANSFORM METHODS - UPSEB SINGLE CIRCUIT
LINE SEQUENTIAL ENERGISATION.

these reactors are introduced, the equations of Section 7.3 get modified as given in the following para.

The matrix $\begin{bmatrix} [A] & [B] \\ [B] & [A] \end{bmatrix}$ in equation (7.1) can be interpreted as the short circuit admittance parameters of the transmission line. The introduction of the shunt admittances $[Y_{sh}]$ at both the ports just adds these admittance values to the self admittance terms of the short circuit admittance parameter matrix, for obtaining the overall short circuit parameters of the system.

Hence equations (7.4) and (7.5), with the reactors introduced, read as follows:

$$\underline{V}_S = \{ [A]+[Y_{sh}]+[Y_s]-[B]([A]+[Y_{sh}])^{-1}[B]\}^{-1} \underline{I}_S \quad (7.8)$$

$$\underline{V}_R = \{ [B]-([A]+[Y_{sh}]+[Y_s])[B]^{-1}([A]+[Y_{sh}])\}^{-1} \underline{I}_S$$
$$(7.9)$$

where $[Y_{sh}] = \begin{bmatrix} 1/(a+j\omega)L_{sh} & 0 & 0 \\ 0 & 1/(a+j\omega)L_{sh} & 0 \\ 0 & 0 & 1/(a+j\omega)L_{sh} \end{bmatrix}$

and L_{sh} is the inductance of the shunt reactor.

7.5.2 Effect of Incorporating Frequency Dependence of Line Parameters:

In order to investigate the effect of frequency dependence of line parameters on the transient overvoltage, produced by the sequential closing of circuit breaker poles,

a single circuit of the double circuit line example,

studied in Chapters 4,5 and 6 is considered. This line

is 150 miles long and has a source inductance of 0.1 H.

The continuous variation of the line parameter values with

frequency are calculated, using the Carson's formulae and

are used in the calculations. The switching instants are

assumed to be 0, 3.3 and 6.7 m.secs. The phase sequence of

pole closing is assumed to be RBY, so that each pole gets

closed when the source voltage of that phase is at peak

value. The comparison of the transient voltage waveforms

at the open receiving end with and without frequency

dependence of line parameters is done in Figure 7.4. From

the waveforms it can be observed that the maximum peak

overvoltage value gets reduced considerably and the

oscillations in the voltage waveforms get smoothened, when

the frequency dependence of line parameters is considered.

A maximum peak of 2.72 p.u. occurs on phase Y at 9.3 m.secs.

for the case without frequency dependence of line parameters;

whereas a maximum peak of -2.36 p.u. occurs on phase B at

5.9 m.secs. for the case with frequency dependence of line

parameters, included. The line is assumed to be uniformly

transposed for both the cases.

7.6 ENERGISATION TRANSIENT ON A DOUBLE CIRCUIT LINE

In this section, the necessary theory for studying

the transient on a double circuit line, due to the sequential

closing of the circuit breaker poles of the second circuit,
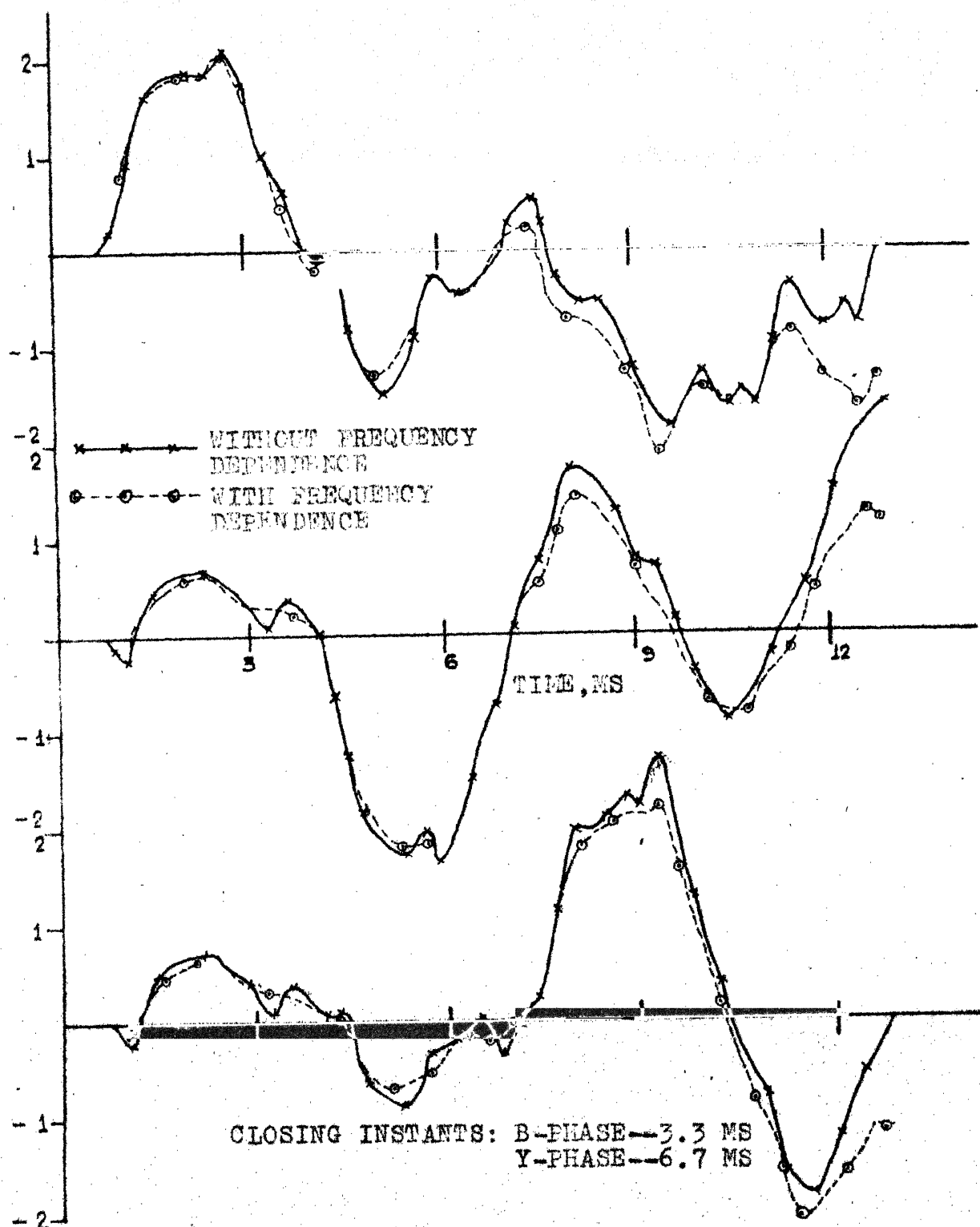
FIGURE 7.4: EFFECT OF FREQUENCY DEPENDENCE OF LINE PARAMETERS ON SEQUENTIAL ENERGISATION TRANSIENTS OF A TRANSPOSED SINGLE CIRCUIT LINE.

when the first circuit is operating in steady state, is developed. Section 5.5 describes the method of calculating the transient due to the closing of the second circuit but for the simultaneous closing of the breaker poles only. For the energisation study, introducing the sequential closing feature also, the principle of superposition can be used, as shown in Section 5.5. But additionally here, the effect of induced voltages on the floating conductors prior to their energisation has to be considered. As described in Section 5.5, the procedure for calculating the second circuit energisation transient of a double circuit line involves

(i) a steady state analysis, assuming the second circuit to remain unenergised and

(ii) the determination of the transient response of the line for a known set of sending end voltage conditions.

It has been shown in Section 5.5.1 that for the case of a transposed double circuit line, the induced voltages on the unenergised conductors, obtained in the above mentioned steady state analysis (step i) will be negligible. Hence the voltages to be injected at the sending end on the second circuit conductors, for which the transient response has to be calculated (in step ii) are the steady state values of voltages of the connected phases at the sending end of the first circuit. The voltages on the first circuit conductors at the sending end for the calculation of the above transient response are zero.

The procedure for the calculation of this transient response at the point of interest (in this case the voltage at the receiving end) on the double circuit line may be described by referring to Figure 7.5.

The general expressions for transformed voltage and current vectors at any point on a transmission line, subsequent to a switching operation, using the same notation as used early in this chapter, are given by

$$\underline{V} = [\mathrm{Cosh}\,\psi x]\underline{K} + [\mathrm{Sinh}\,\psi x]\underline{M} \tag{7.10}$$

$$\underline{I} = -[Z]^{-1}[\psi]\,\{[\mathrm{Cosh}\,\psi x]\underline{M} + [\mathrm{Sinh}\,\psi x]\underline{K}\} \tag{7.11}$$

where $\underline{K}$ and $\underline{M}$ are constants, determined by the boundary conditions, existing at the ends of the line.

In the interval $0 < t < T_1$, only the first phase of the second circuit is closed. Hence in this interval, the voltage on the first phase alone gets the value of the injected voltage and the other conditions, to be forced at the sending end are zero currents on the two unenergised conductors of the second circuit, in addition to the zero voltage conditions on the first circuit conductors.

By forcing the known voltage conditions at the sending end, the first four components of $\underline{K}$ in equation (7.10) are determined as follows:

$$K_1 = 0 \;\; ; \;\; K_2 = 0 \;\; ; \;\; K_3 = 0 \tag{7.12}$$

$K_4 = E_{s1}$, the modified Fourier transform of the voltage injected to the first phase of the second circuit.
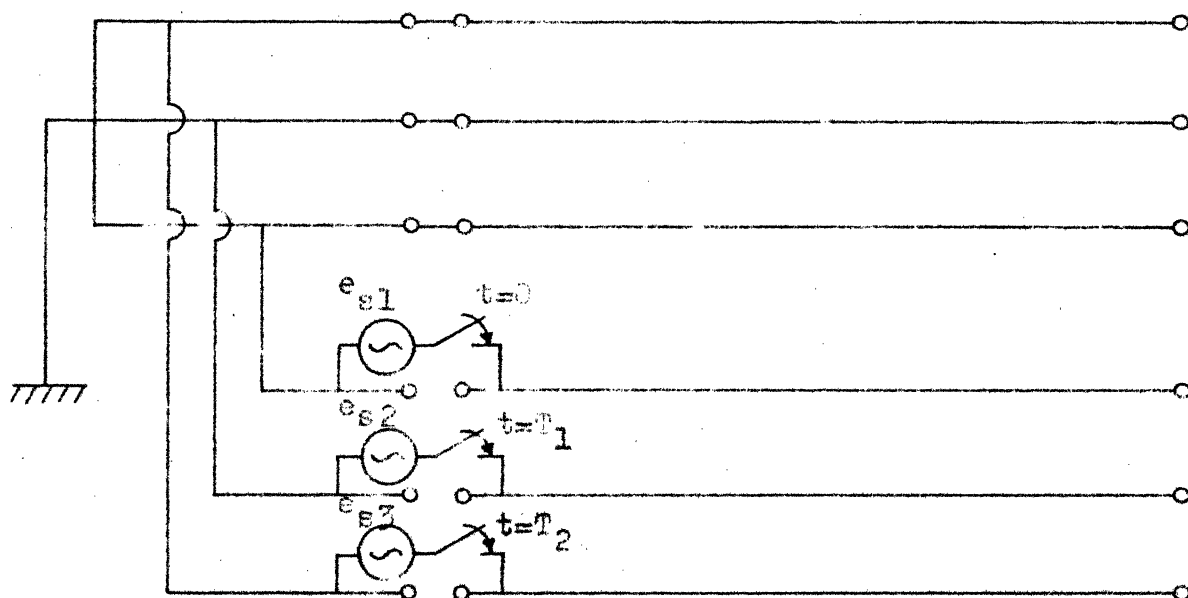
FIGURE 7.5: SYSTEM DIAGRAM FOR CALCULATION OF TRANSIENT
            RESPONSE FOR INJECTED VOLTAGE — SEQUENTIAL
            CLOSING.

Denoting the elements of matrices by double subscripted lower case letters and the matrix $[Z]^{-1}[\psi]$ by $[U]$, forcing of the zero current conditions on the two unenergised phases of the second circuit, make equation (7.11) take the form

$$- \begin{bmatrix} u(5,1) & u(5,2) & \dots & u(5,6) \\ u(6,1) & u(6,2) & \dots & u(6,6) \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (7.13)$$

The open receiving end condition gives rise to another equation

$$\underline{M} = -[\text{Tanh}\,\psi\ell]\,\underline{K} \qquad (7.14)$$

Replacing $\underline{M}$ in (7.13) by (7.14), we get

$$\begin{bmatrix} u(5,1) & u(5,2) & \dots & u(5,6) \\ u(6,1) & u(6,2) & \dots & u(6,6) \end{bmatrix} [\text{Tanh}\,\psi\ell]\,\underline{K} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (7.15)$$

Denoting

$$\begin{bmatrix} u(5,1) & u(5,2) & \dots & u(5,6) \\ u(6,1) & u(6,2) & \dots & u(6,6) \end{bmatrix} [\text{Tanh}\,\psi\ell] \text{ by } [UT]$$

and substituting for known components of $\underline{K}$ by equation (7.12), (7.15) becomes

$$\begin{bmatrix} ut(1,1) & ut(1,2) & \dots & ut(1,6) \\ ut(2,1) & ut(2,2) & \dots & ut(2,6) \end{bmatrix} \begin{bmatrix} 0 \\ E_{s1} \\ K_4 \\ K_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (7.16)$$

where $\quad \underline{O} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

From (7.16), K_4 and K_5 can be expressed in terms of known quantities as follows:

$$\begin{bmatrix} K_4 \\ K_5 \end{bmatrix} = - \begin{bmatrix} ut(1,5) & ut(1,6) \\ ut(2,5) & ut(2,6) \end{bmatrix}^{-1} \begin{bmatrix} ut(1,4) \\ ut(2,4) \end{bmatrix} E_{sl} \qquad (7.17)$$

K_4 and K_5 are nothing but the transforms of the induced voltages at the sending end on the second and third phases of the second circuit.

These transforms of the induced voltages on the floating conductors are to be transferred to time domain by performing the numerical integration. For continuing the calculations, when the second and the third poles get closed, a numerically evaluated modified Fourier transform of these induced voltages on these floating conductors during the interval when they are open should be added to the analytically known transforms of the second and third phase injected voltages of the second circuit, which come into the calculations from instants T_1 and T_2 respectively.

At the instant T_1, the second pole gets closed and now the conditions to be forced at the sending end are known voltages on first and second phase conductors and zero current on the third unenergised phase conductor of the

second circuit, in addition to zero voltage conditions on the three phases of the first circuit.

The known transformed voltages at the sending end determine the first five components of the vector $\underline{K}$ now as follows:

$$K_1 = 0 \; ; \quad K_2 = 0 \; ; \quad K_3 = 0 \; ; \quad K_4 = E_{s1}$$
$$K_5 = E_{s2} + V_{s2}$$

(7.18)

where E_{s2} is the analytically known modified Fourier transform of the injected voltage on the second phase of the second circuit (which starts from instant T_1) and V_{s2} is the numerically evaluated modified Fourier transform of the induced voltage on the second phase of the second circuit in the interval $0 < t < T_1$.

The zero current condition on the third phase of the second circuit gives rise to the equation

$$[ut(1,1) \quad ut(1,2) \quad \ldots \quad ut(1,6)] \begin{bmatrix} \underline{0} \\ E_{s1} \\ (E_{s2}+V_{s2}) \\ K_6 \end{bmatrix} = 0 \quad (7.19)$$

where [UT] is the same as used earlier and

$$\underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From equation (7.19), the unknown K_6 can be expressed in terms of known quantities as follows:

$$K_6 = -[ut(1,4) \ E_{s1} + ut(1,5)(E_{s2} + V_{s2})]/ut(1,6) \quad (7.20)$$

K_6 so obtained is the transform of the induced voltage on the third phase of the second circuit, valid in the time interval $0 < t < T_2$. The addition of the numerically evaluated modified Fourier transform of this induced voltage in this interval to the analytically known transform of the third phase injected voltage is essential for carrying on the transient calculations after the instant T_2.

At instant T_2, the third pole closes and now the transformed voltages on all the phases at the sending end, valid in the entire time range are known.

Hence for the open receiving end condition, an expression for the transformed voltage at the receiving end can be derived using the general expressions (7.10) and (7.11) as follows:

$$\underline{V}_R = [\text{Sech}\psi\ell] \ \underline{V}_s$$

where $\underline{V}_s$ is the known transformed voltage vector at the sending end.

A numerical integration of this transformed voltage $\underline{V}_R$ to time domain gives the desired transient response.

This transient response, when added to the steady
state response, calculated assuming the second circuit to
be unenergised, yields the total response.

7.7 COMPUTATIONAL RESULTS
7.7.1 Double Circuit Versus Single Circuit Lines:

The same double circuit line example, considered in
Chapters 5 and 6 is taken up and the transient overvoltage
at the open receiving end, resulting from the sequential
closing of circuit breaker poles is evaluated by the procedure
described in the last section. Next the presence of the first
circuit is ignored and the receiving end transient voltage,
resulting due to the injection of the steady state voltage,
calculated at the sending end of the first circuit in the
steady state analysis, is computed. The line is assumed to
be transposed and the closing instants of the phases are
taken to be 0, 3.3 and 6.7 m.secs. for both the cases. The
phase sequence of breaker pole closing is assumed to be
R'B'Y'. The 50 Hz parameters of the line are used in the
calculations. The open receiving end voltage waveforms for
the single and double circuit representations are compared
in Figure 7.6. A maximum peak of 2.505 p.u. is observed to
occur on phase Y' at 8.3 m.secs. for the case of double
circuit representation; whereas the maximum peak for the case
of single circuit representation is -2.9 p.u. on phase B' at
4.8 m.secs. Noticing the wide variation in the voltage
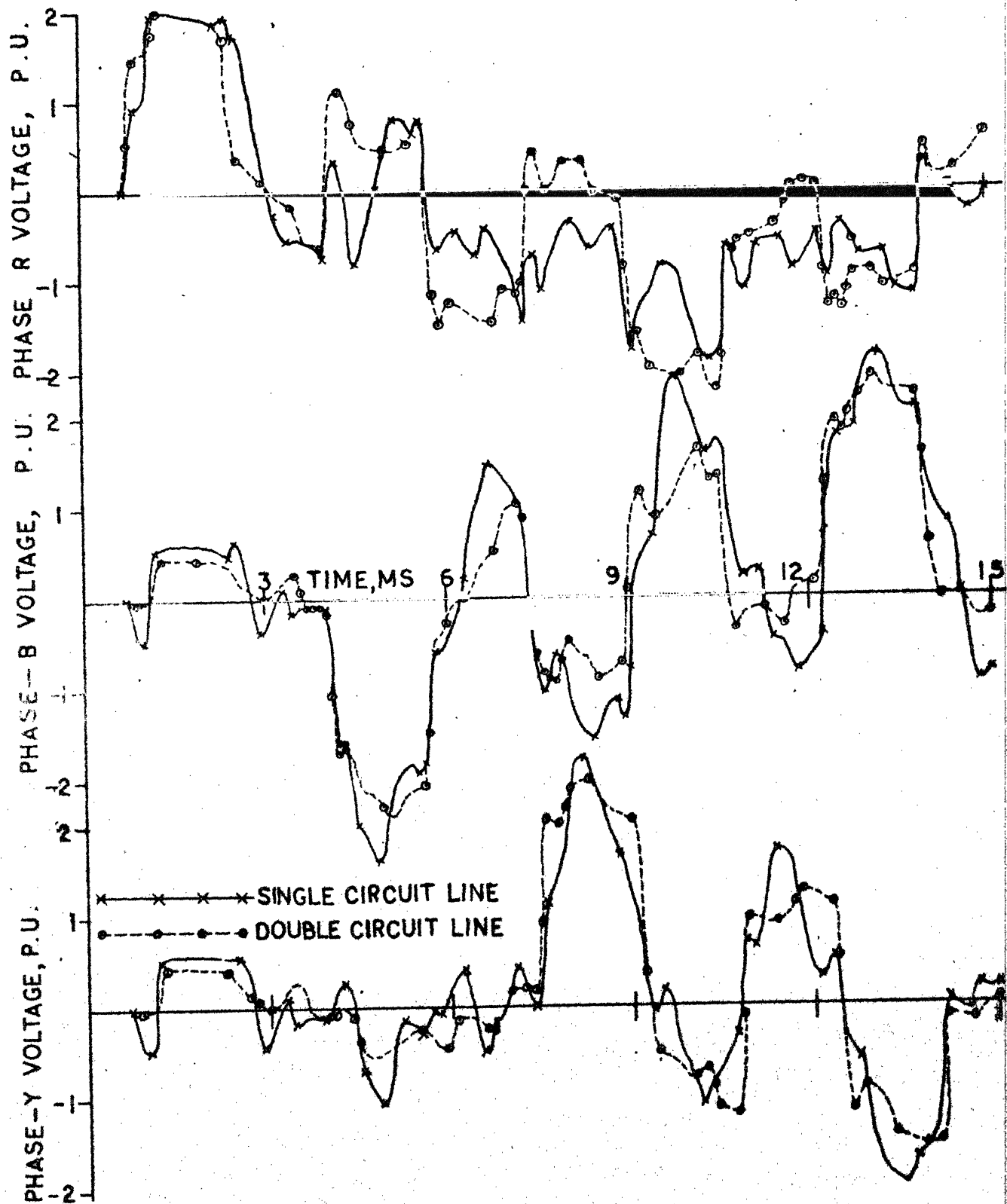
FIGURE 7.6: COMPARISON OF VOLTAGE WAVEFORMS AT RECEIVING
END FOLLOWING SEQUENTIAL CLOSING FOR TRANSPOSED
SINGLE AND DOUBLE CIRCUIT LINES.

waveforms and in the maximum peak values(about 16 percent difference) of both the cases, it may be concluded that the double circuit modelling is essential for the energisation transient study.

7.7.2 Transposed Versus Untransposed Double Circuit Lines:

Next the energisation transient of the same example for the untransposed configuration of the line is evaluated. The eigenvalue, eigenvector analysis at each frequency in the integration range is performed for the double circuit line to arrive at the modal transformation matrix for using in these calculations. The voltage waveforms at the open receiving end for the transposed and untransposed configurations of the line are compared in Figure 7.7. Considerable difference in waveshapes is observed for the two configurations as there are six distinct modes and velocities of propagation for the case of untransposed line, whereas for the transposed line there are only three such distinct modes and velocities of propagation. A maximum peak voltage of 2.505 p.u. is observed to occur on phase Y' at 8.3 m.secs. for the transposed line. The maximum peak for the untransposed line is -2.46 p.u. and it occurs on phase B' at 4.95 m.secs.

7.7.3 Effect of Frequency Dependence of Line Parameters:

The effect of continuous variation of line parameter values with frequency on the transient overvoltage waveforms
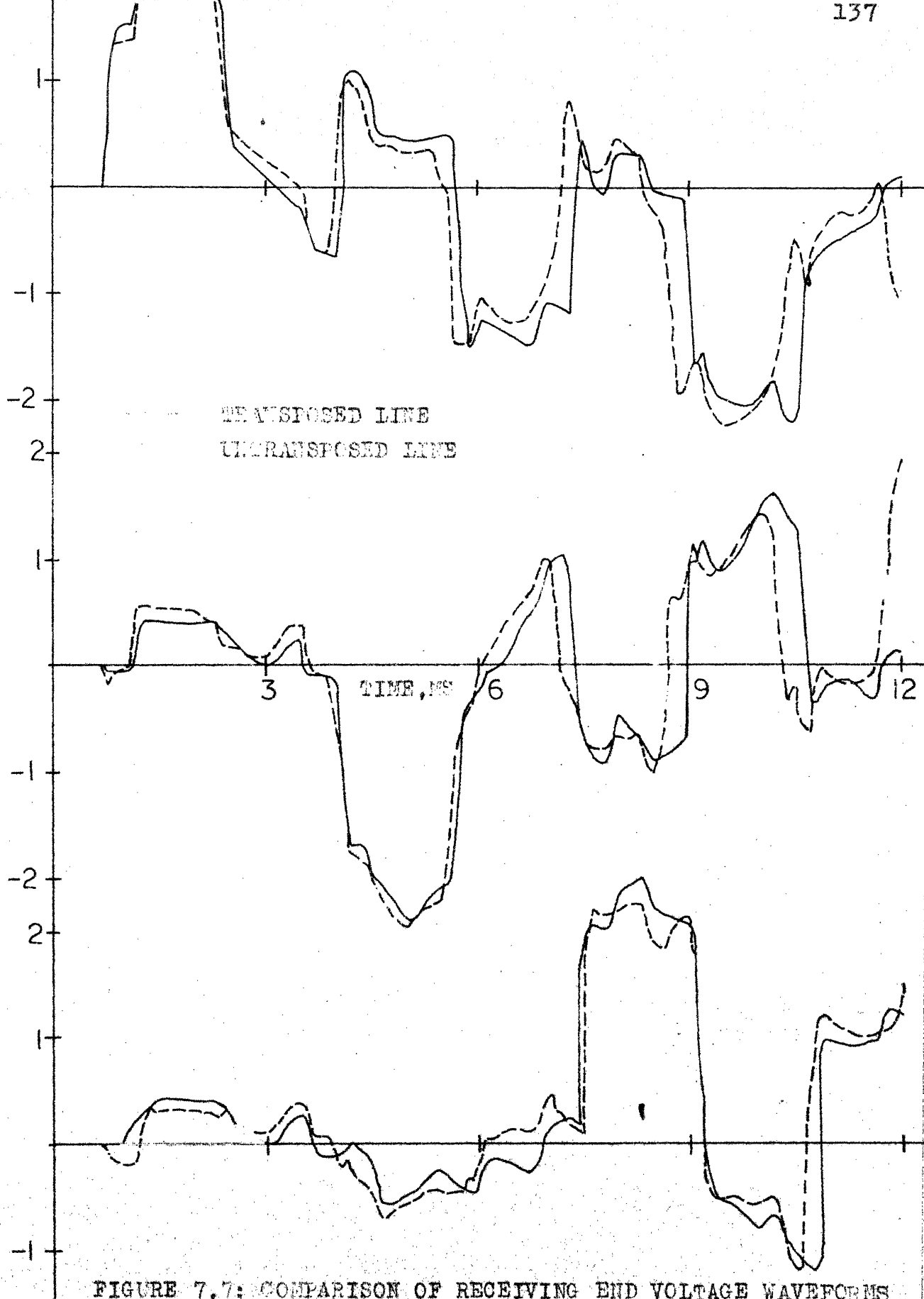
FIGURE 7.7: COMPARISON OF RECEIVING END VOLTAGE WAVEFORMS
FOLLOWING SEQUENTIAL CLOSING FOR TRANSPOSED
AND UNTRANSPOSED LINES.

due to sequential closing of transposed and untransposed double circuit lines is next examined. The line parameters, calculated at each frequency using Carson's formulae are used in the calculations. The resulting transient voltage waveforms at the open receiving end for the same example, with and without frequency dependence of line parameters incorporated for transposed and untransposed lines are compared in Figures 7.8 and 7.9 respectively. Again the closing instants of the phases are assumed to be 0, 3.3, 6.7 m.secs. The phase sequence of closing is taken as R'B'Y'. For the transposed configuration of the line, without considering the frequency dependence of line parameters, the maximum peak overvoltage (2.505 p.u.) occurs on phase Y' at 8.3 m.secs. When the frequency dependence of line parameters is considered the maximum peak value reduces to 2.21 p.u. (about 14 percent reduction) and this occurs at 8.1 m.secs. on phase Y'. The corresponding peak values for the untransposed configuration of the line without and with frequency dependence of line parameters, incorporated, are -2.46 p.u. (on phase B' at 4.95 m.secs.) and -2.28 p.u. (on phase B' at 4.95 m.secs.) respectively. There is also a smoothening of oscillations in the waveforms when frequency dependence of line parameters is included for both the configurations of the line.
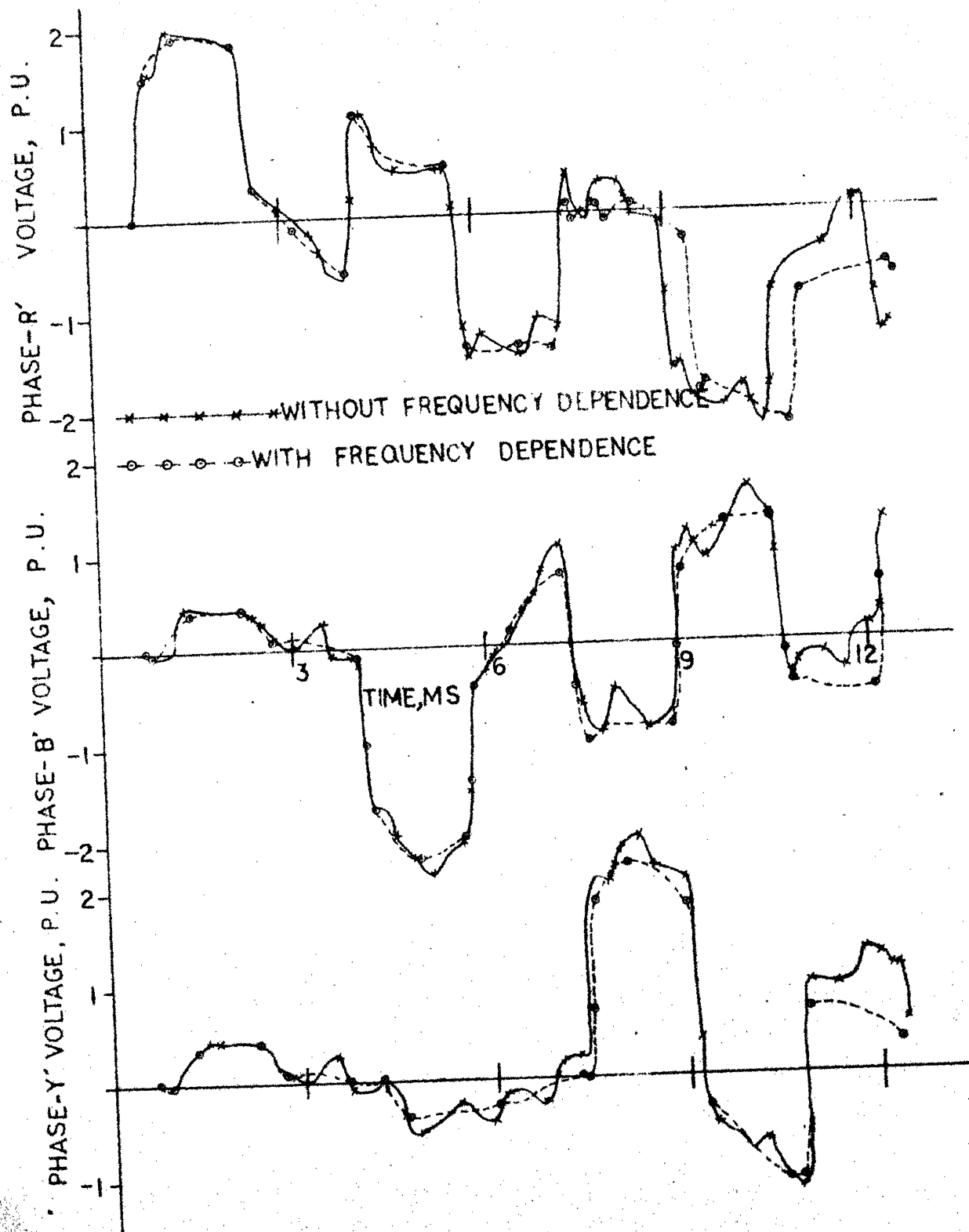
FIGURE 7.8: EFFECT OF FREQUENCY DEPENDENCE OF
LINE PARAMETERS ON VOLTAGE TRANSIENTS
DUE TO SEQUENTIAL CLOSING FOR TRANSPOSED
DOUBLE CIRCUIT LINE.
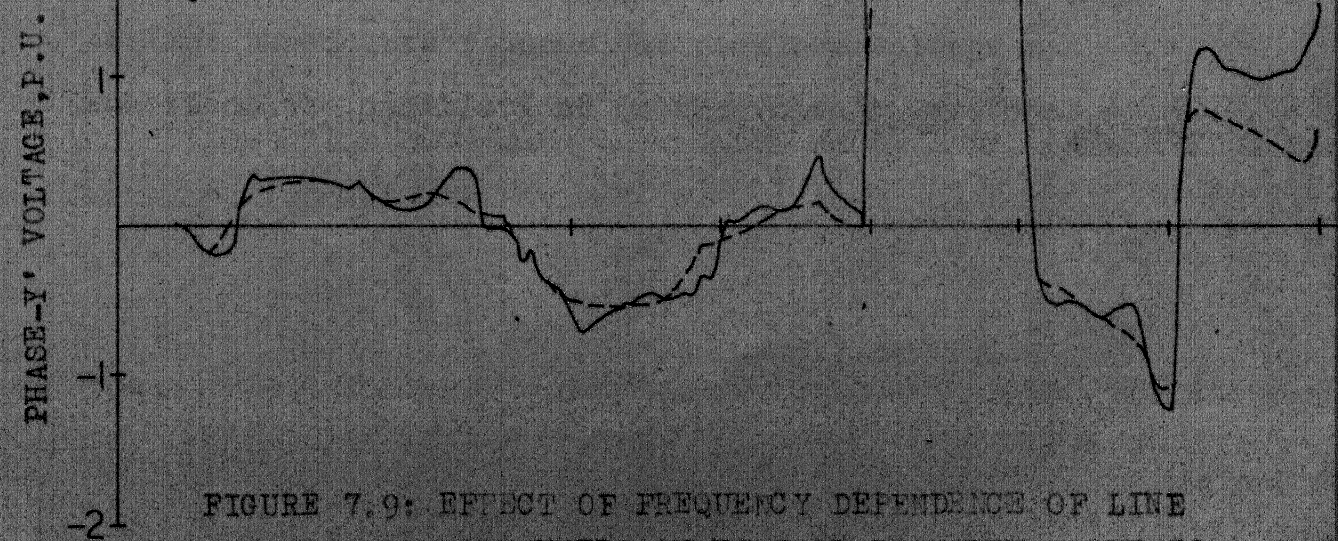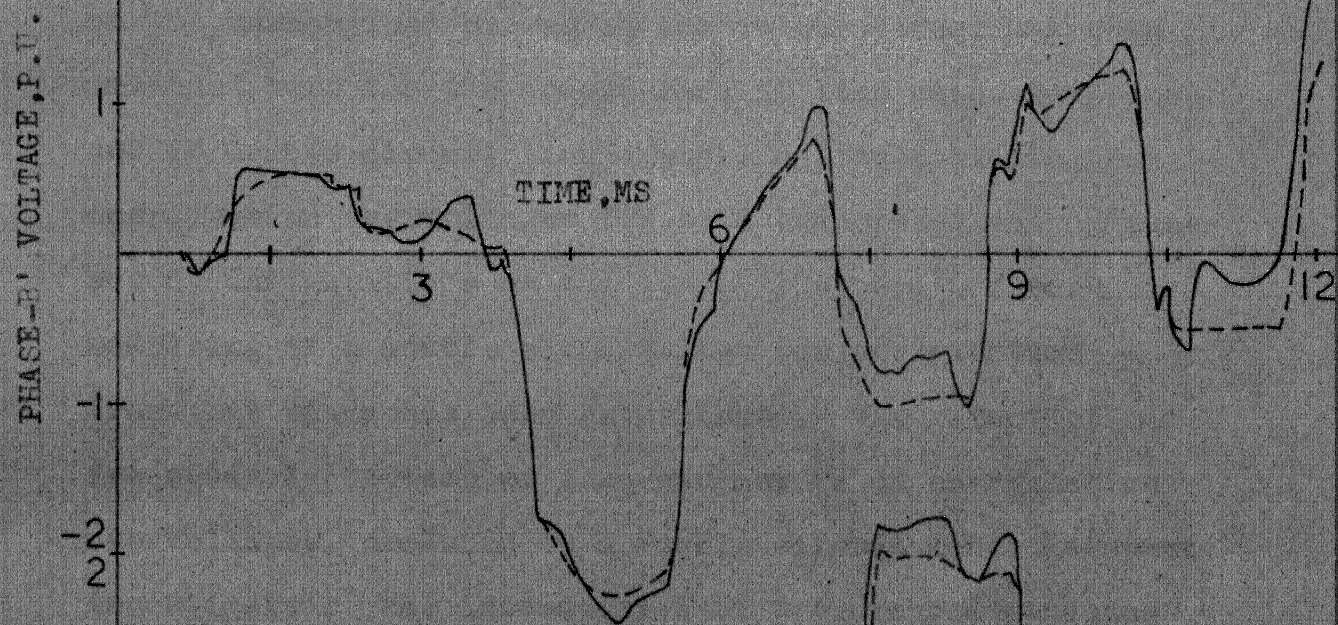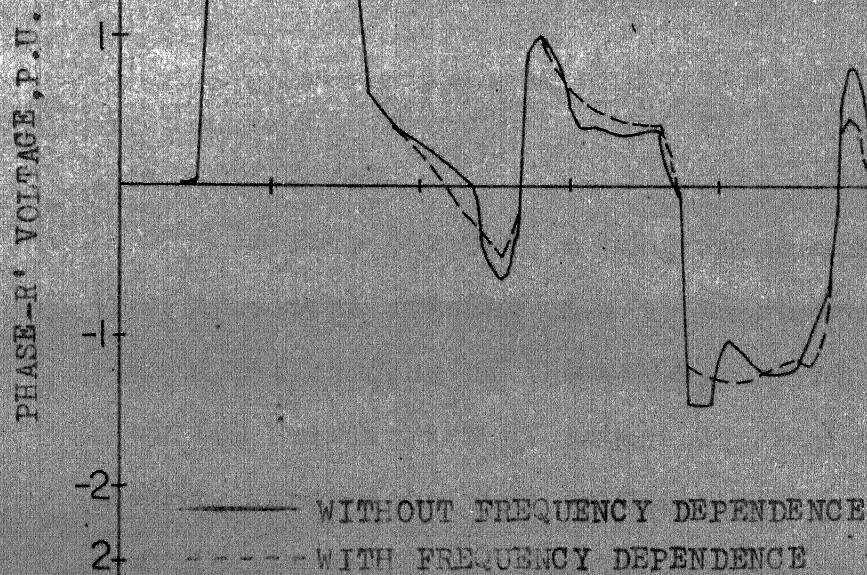
FIGURE 7.9: EFFECT OF FREQUENCY DEPENDENCE OF LINE
PARAMETERS ON VOLTAGE TRANSIENTS DUE TO
SEQUENTIAL CLOSING FOR UNTRANSPOSED
DOUBLE CIRCUIT LINE.

7.8 CONCLUSIONS

In this chapter, a program to compute the energisation transient arising on a transmission line, due to sequential closing of the circuit breaker poles, by the modified Fourier transform method, has been described. The method, first developed by Wedepohl and Mohamed, using numerically evaluated Fourier transform of induced voltages on the unenergised phases in the calculations, has been extended here for the double circuit line case. A typical 400 KV double circuit line example has been studied for energisation overvoltages for both transposed and untransposed configurations of the line. The need for the modelling of a double circuit line for energisation transient study has been established. The effect of frequency dependence of line parameters on energisation overvoltages, produced on a double circuit line, has been investigated. The introduction of frequency dependence of line parameters reduces the peak overvoltage and smoothens the oscillations in the voltage waveforms.

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 CONCLUSIONS

8.1.1 : The study of electrical transients on EHV
power systems has gained great importance, particularly in
view of the increasing operating voltage of the transmission
systems. The cost of insulation forms a major portion of
the total cost of a transmission system. Especially as the
system voltage increases to EHV level, the insulation cost
of the line increases at a very rapid rate. While doing
these planning studies, if the models of the transmission
line and other equipments that are employed in the transient
calculations are not very accurate, it is necessary to use
a high factor of safety to take care of the likelihood of
the consequent margin of error in the calculated values.
This ultimately reflects in higher cost of insulation. If
this factor of safety and insulation costs are to be reduced,
it can be done only if the calculated results predict the
system performance with a high degree of accuracy. To achieve
this, it is necessary to employ accurate models for all the
components of a system while doing the required studies.

Double circuit lines are common features of any
power system. However, digital calculation of transients
on double circuit lines have not been reported. Nevertheless
a few TNA studies for double circuit lines have been reported

as acknowledged in Section 5.1. An accurate mathematical model of a double circuit line suitable for studying the electrical transients arising on it has been proposed in this thesis. Further, this model is made use of to study the energisation and fault transients on a typical 400 KV double circuit line. This illustrates the use of the above method and also shows that it is essential to represent the double circuit lines adequately in any transient studies.

There are various methods of calculating the over-voltage transients that are available in the literature. These methods and their special features are discussed in Chapter 1. Among these, the modified Fourier transform method is regarded as one of the more accurate methods, because it considers all the line parameters to be distributed, can incorporate the effect of frequency dependence of line parameters with ease and can also handle untransposed lines. Therefore the proposed model in this thesis largely refers to the modified Fourier transform method. In one of the chapters, it has also been applied to Uram and Miller's method. Similarly, it can be extended to other methods, if desired.

8.1.2: A mathematical model of a double circuit line, suitable for transient overvoltage studies has been developed in Chapter 3. It has been shown in Section 3.2.1

that there exists a constant modal transformation matrix
for the case of a transposed double circuit line, whose
conductors are situated at a mirror symmetric position
with respect to a vertical plane. This transformation
holds good at each frequency in the numerical integration
range. Thus, for calculations of switching overvoltages
involving transposed double circuit lines considerable
saving in computation time can be achieved. On the other
hand, in the case of untransposed double circuit lines,
the eigenvalue,eigenvector analyses of two 3x3 matrices are
to be performed, at each frequency, to arrive at the modal
transformation matrix.

8.1.3: The transient overvoltages, arising on the
sound phases of a double circuit line due to the initiation
of a single line to ground fault on one of the circuits,
have been studied in Chapter 4. Necessity for the study
of the transients due to the initiation and clearing of
fault in a system arises when effective methods are
adopted for controlling the overvoltages, caused by
energisation and re-energisation of the line. At present,
as there are very effective methods, such as multistep
resistance closing,controlling the closing instants of the
individual poles of the circuit breaker etc. available to
limit the energisation and re-energisation overvoltages, the
fault initiation and clearing transients may dominate the
scene. These fault transients have been studied by various

authors in the past for single circuit lines using various digital methods. However, an accurate analysis of these fault transients on double circuit lines has not been reported. Moreover, in all the above studies, the lines were assumed to be transposed and the frequency dependence of line parameters have not been taken into account since they used lattice diagram or Laplace transform approach. In this thesis the modified Fourier transform method has been employed for these studies. Further, this has also been extended to the double circuit line model that has been proposed. A typical 400 KV double circuit line example has been considered. The transient voltage waveforms on the sound phases, following a single line to ground fault at mid span on the line, have been computed for both the transposed and untransposed configurations of the line. As shown in Figure 4.5, the maximum peak voltage in the case of untransposed line is observed to be nearly 10 percent higher than that of the transposed case.

In the above set up the presence of the second circuit is ignored for calculation of the same transients, thus representing a single circuit line. In this case fault is considered on one of the phases and the resulting transients on the remaining sound phases are evaluated. The voltage waveforms on the sound phases at the fault point for the double and single circuit representations have been shown

in Figure 4.4. The voltage waveforms for the case of the
double circuit line clearly exhibit more high frequency
oscillations than those for the case of the single circuit
line. This seems to be due to the fact that there are
three distinct modes of propagation with three distinct
velocities of propagation in the case of transposed double
circuit line; whereas for the case of a transposed single
circuit line, there are only two such distinct modes and
velocities of propagation. A maximum peak of -1.78 p.u.
is observed to occur on phase-c' of the second circuit
for the double circuit line representation. For the
single circuit line, the maximum peak of -1.98 p.u.
(10 percent more) occurs on phase-c. This order of
difference between the peak voltages for the single and
double circuit representations shows that it is important
to represent the double circuit line adequately. The
effect of frequency dependence of line parameters on the
fault transient overvoltages for both the transposed and
untransposed configurations of the transmission line has
also been investigated as shown in Section 4.6.3. The
inclusion of the frequency dependence of line parameters
is observed to reduce the maximum peak voltage by nearly
11 percent and 13 percent (see Figures 4.6 and 4.7) for
the cases of transposed and untransposed lines respectively.
There is also a smoothening of oscillations in the transient
voltage waveforms when frequency dependence is included in

the calculations. The fault transients have been evaluated
for various off-centre fault locations as shown in Table 4.2.
The mid span fault is found to give rise to the maximum peak
overvoltage.

8.1.4: The modified Fourier transform method has been
applied to analyse the energising transients on double circuit
lines in Chapter 5. Algorithms have been developed to study
the transients due to the closing of the first circuit with
the second circuit unenergised, as well as for the transients
due to the closing of the second circuit when the first
circuit is operating in steady state. Here all the three
poles of the circuit breaker are assumed to close simultane-
ously during energisation. A typical 400 KV double circuit
line, open at the receiving end has been considered and
both the above mentioned energising transients at the
receiving end have been calculated. The energising transient
for the double circuit line representation has been compared
with the transient caused by the energisation of the corres-
ponding single circuit line (ignoring the presence of the
other circuit). For the untransposed configuration of the
line, a difference of 20 percent in the peak transient
voltages is observed for the single and double circuit
representations (Figure 5.6). Apart from such a consider-
able difference in peak overvoltages, the voltage waveforms
for these two representations also differ widely in shape.
This may be attributed to the reason that in the case of

untransposed double circuit lines there are six distinct modes and velocities of propagation; whereas there are only three such distinct modes and velocities of propagation in the case of untransposed single circuit lines.

The effect of inclusion of the frequency dependence of line parameters is observed to be insignificant for the case of the simultaneous closing of the breaker poles. This appears to be due to the absence of the zero sequence currents for simultaneous closing of the circuit breaker poles and only the zero sequence components of the resistance and inductance of the line (ground return path taken into account) vary considerably with frequency.

8.1.5: In Chapter 6, the Uram and Miller's method has been extended to study the energising transients on double circuit lines. Uram and Miller's method is a time domain method, in which by making some assumptions, an incremental solution of the transmission line equations is made possible. The existence of a frequency independent modal transformation matrix is necessary for the application of this method. Such a transformation is shown to exist for the transposed double circuit line in Section 3.2.1. The same double circuit line example as in Chapter 5 has been considered. The transients at the open receiving end, following the energisation of the first circuit with the second circuit unenergised as well as that due to the closing of the second circuit when the first circuit is

operating in steady state have been computed. The results
obtained by this method are found to be closely matching with
those obtained using the modified Fourier transform method.
This serves as an important check on the validity and
accuracy of the method proposed here as well as the two
computer programs developed for this purpose. The Uram
and Miller's method takes much less computer time but can
handle only transposed lines and cannot take into account
the frequency dependence of line parameters directly.

8.1.6: So far in all the studies all the three poles
of a circuit breaker are assumed to operate simultaneously.
However, in reality, there may be a difference of a few
m.secs. in the operation of different poles of the circuit
breaker. The incorporation of the nonsimultaneous closing
feature of the circuit breaker poles in the modified Fourier
transform method of calculating energisation transients
has been described in Chapter 7. The method has been
extended to evaluate the transient arising due to sequential
energisation of the second circuit of a double circuit line
when the first circuit is operating in steady state. The
same 400 KV double circuit line example as of Chapters 5
and 6 has been studied. When the sequential closing of the
breaker poles is incorporated in the study, for the transposed
configuration of the line, the peak voltage is 16 percent
lower in the case of a double circuit line as compared to
that of the corresponding single circuit line (Figure 7.6).

It may be noted here that there was no difference in the peak values for transposed single and double circuit lines when the breaker poles are assumed to close simultaneously (see Section 5.6.2). The need for the modelling of the double circuit line has been established by comparing the transient arising on the second circuit obtained by ignoring the presence of the first circuit with that obtained by the double circuit representation.

The transient voltages at the receiving end following sequential closing for the transposed and un-transposed configurations of the double circuit lines have been studied. A considerable difference in wave-shapes has been observed for the two configurations as shown in Figure 7.7.

The effect of inclusion of frequency dependence of line parameters on the transient overvoltages arising on the double circuit line following sequential closing has also been investigated. The introduction of the frequency dependence of line parameters is observed to reduce peak overvoltages by 14 percent and 8 percent for the transposed and untransposed configurations of the line respectively for the example considered. There is also a smoothening of oscillations in the voltage waveforms.

In this thesis, the transient arising due to re-energisation of a double circuit line on trapped charges

has not been considered. However, the method is general enough and this aspect may be studied in future. In the various examples considered in this thesis the source sides have been represented by lumped impedances. The values of these source impedances have been derived from the system short circuit levels. This simplified model for the source is somewhat approximate. However, here the aim is just to illustrate the application of the modified Fourier transform method for transient analysis on double circuit lines and hence source sides have not been rigorously represented. However, there is no limitation for the applicability of the method proposed here for a detailed source representation. The other lines terminating at the ends of the line under consideration, can be represented quite accurately by shunt elements at these buses, the values of which can be obtained by finding out the equivalent admittances offered by these lines at these buses, taking into consideration the distributed nature of these lines and the terminal conditions existing at the remote ends of these lines.

In addition to the above the following suggestions are made for future extension of this work.

1. Computation of transient arising due to the re-energisation of a double circuit line on trapped charge.

2. Fault clearing overvoltages and transient recovery voltages evaluation for a system consisting of double circuit lines.

3. Development of a general purpose program, using modified Fourier transform method to accommodate general source side network.

4. Studies of transients in systems, consisting of non-linearities such as transformer saturation, lightning arresters etc. using modified Fourier transform method for the line.

5. Studies of overvoltage transients on double circuit lines with series capacitor compensation.

8.2 SUMMARY

We have seen that in the case of a typical 400 KV line, considerable difference in calculated results is obtained when the line is represented by a single circuit or a double circuit model. In particular, the following results are very significant.

1. The maximum peak overvoltage arising on sound phases following a single line to ground fault for the transposed double circuit representation is -1.78 p.u. The corresponding maximum peak for the transposed single circuit representation is -1.98 p.u.

2. For the transposed configuration of the double circuit line, maximum peak voltage due to fault initiation is -1.78 p.u. For the untransposed configuration, the corresponding maximum peak is -1.97 p.u.

3. When frequency dependence of line parameters is consi-
    dered, the maximum peak voltage due to fault initiation
    reduces by 11 percent and 13 percent for the transposed
    and untransposed configurations of the double circuit
    line respectively.

4. The maximum peak voltage due to simultaneous breaker
    pole closing of the first circuit of a transposed
    double circuit line is -2.45 p.u. and occurs on phase-a
    at 9.2 m.secs. The corresponding maximum peak for
    the untransposed double circuit line is 2.24 p.u. and
    this occurs on phase-c at 12.8 m.secs.

5. The maximum peak voltage due to simultaneous breaker
    pole closing of the second circuit of the transposed
    double circuit line is 2.02 p.u. and occurs on phase a'
    at 1.02 m.secs. The corresponding maximum peak for
    the case of untransposed double circuit line is
    2.23 p.u. and occurs on phase a' at 0.9 m.secs.

6. With sequential closing of circuit breaker poles
    incorporated, the maximum peak voltage due to second
    circuit energisation for transposed double circuit
    line is 2.505 p.u. and occurs on phase Y' at 8.3 m.secs.
    The maximum peak value for the corresponding single
    circuit line is -2.9 p.u. and occurs on phase B' at
    4.8 m.secs.

7. A maximum peak voltage of 2.505 p.u. is observed to
    occur on phase Y' at 8.3 m.secs. for sequential second

circuit energisation of the transposed double circuit line; whereas the corresponding maximum peak for untransposed double circuit line is -2.46 p.u. on phase B' at 4.95 m.secs.

8. Reduction in peak values of 14 percent and 8 percent is observed in the transients due to second circuit sequential energisation for transposed and untransposed double circuit lines respectively, when frequency dependence of line parameters is considered.

The order of magnitude of the error in the above cases is such that it appears vital to consider the following while doing any transient studies.

a. Double circuit lines should be represented adequately.

b. Transposition of the lines or otherwise, should be taken into account.

c. Frequency dependence of line parameters can be ignored only when it is justified specifically.

d. Non-simultaneous operation of the circuit breaker poles should be considered in these calculations.

In the opinion of the author, optimum costs of insulation of a transmission line can be achieved only when all the above factors are taken into account. These have been implemented in the computer programmes developed by the author and therefore the above suggestions are quite feasible. The validity of these suggestions is well borne out by the study of a typical 400 KV line that has been

carried out throughout. In particular power systems, the errors due to the various approximations may be even higher than those reported here. Therefore, the importance of accurate representation of the components of the system in transient calculations cannot be under emphasised. Methods to incorporate all the above suggestions have been successfully implemented in the procedures proposed in this thesis.

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APPENDIX   A

SYSTEM DATA

Length of the line   240 Kms.

Source inductance   0.1 H/phase

Earth conductivity   100 Ohm-m.

50 Hz line parameter matrices for the untransposed configuration are as follows:

$$R = \begin{bmatrix} 0.079658 & 0.049180 & 0.048136 \\ 0.049180 & 0.078826 & 0.048053 \\ 0.048136 & 0.048053 & 0.077874 \end{bmatrix} \times 0.62 \text{ ohm/Km.}$$

$$L = \begin{bmatrix} 0.002493 & 0.001132 & 0.000829 \\ 0.001132 & 0.002434 & 0.000979 \\ 0.000829 & 0.000979 & 0.002291 \end{bmatrix} \times 0.62 \text{ H/Km.}$$

$$C = \begin{bmatrix} 1.89230 & -0.35843 & -0.10716 \\ -0.35843 & 1.81560 & -0.34268 \\ -0.10716 & -0.34268 & 1.74110 \end{bmatrix} \times 0.62 \times 10^{-8} \text{ f/Km.}$$

For the case of the tranposed line, the line parameters are obtained by finding the average values of the diagonal and off-diagonal elements of these matrices separately, corresponding to the untransposed case.

APPENDIX B

LINE PARAMETERS CALCULATION - CARSON'S FORMULAE

Self impedance of conductor $= r_c + 0.00159f$

$$+ j0.004657f \log_{10} \frac{2160\sqrt{(\rho/f)}}{GMR}$$

Mutual impedance between two conductors $= 0.00159f$

$$+ j0.004657f \log_{10} \frac{2160\sqrt{(\rho/f)}}{d_{ab}}$$

where r_c = resistance of the conductor per mile

f = frequency in Hz

ρ = earth resistivity in ohm-m

GMR = geometric mean radius of the conductor in feet

d_{ab} = distance between the conductors in feet.

APPENDIX C

SYSTEM DATA - BOONYUBOL'S EXAMPLE

Positive sequence resistance of the line = 0.03 ohm/mile

Positive sequence inductance of the line = 1.56 mH/mile

Positive sequence capacitance of the line = 0.0189 μf/mile

Zero sequence resistance of the line $\qquad$ = 0.316 ohm/mile

Zero sequence inductance of the line $\qquad$ = 5.76 mH/mile

Zero sequence capacitance of the line $\qquad$ = 0.0138 μf/mile

Frequency of supply $\qquad$ = 60 Hz

Length of the line $\qquad$ = 180 miles

The fault is simulated at mid span.

The source impedance has been taken as zero.

The fault has been assumed to occur on phase 'a' at the instant when phase 'a' voltage is at positive peak.

APPENDIX D

SYSTEM DATA FOR FAULT STUDIES

Length of the line   480 Kms.

The line is fed on both the ends by similar sources, which are purely inductive of value 0.05 H.

Earth conductivity   = 100 ohm-m.

The computed line parameter matrices at 50 Hz for the untransposed configuration are given below. As these matrices are symmetric, only the lower triangular elements are given.

$$
[R] = \begin{bmatrix}
0.079658 \\
0.049180 & 0.078826 \\
0.048136 & 0.048053 & 0.077874 \\
0.049658 & 0.049180 & 0.048136 & 0.079658 \\
0.049180 & 0.048826 & 0.048053 & 0.049180 & 0.078826 \\
0.048136 & 0.048053 & 0.047874 & 0.048136 & 0.048053 & 0.077874
\end{bmatrix}
$$

x 0.62 ohm/Km.

$$
[L] = \begin{bmatrix}
0.002493 \\
0.001132 & 0.002434 \\
0.000829 & 0.000979 & 0.002291 \\
0.000972 & 0.000876 & 0.000754 & 0.002493 \\
0.000876 & 0.000848 & 0.000786 & 0.001132 & 0.002434 \\
0.000754 & 0.000786 & 0.000829 & 0.000829 & 0.000979 & 0.002291
\end{bmatrix}
$$

x 0.62 H/Km.

$$[C] = \begin{bmatrix} 1.89230 \\ -0.35843 & 1.81560 \\ -0.10716 & -0.34268 & 1.74110 \\ -0.12082 & -0.08571 & -0.06246 & 1.89230 \\ -0.08571 & -0.10012 & -0.12187 & -0.35843 & 1.81560 \\ -0.06246 & -0.12187 & -0.26750 & -0.10716 & -0.34268 & 1.74110 \end{bmatrix}$$

$$\times \; 0.62 \times 10^{-8} \; f/Km.$$

For the transposed configuration, the self elements
of the line parameter matrices are obtained by taking the
average value of the self elements of the corresponding
parameter matrices of the untransposed case, given above.
Similarly, the mutual elements between the phases of
each circuit and the mutual elements between the two
circuits for the transposed line are the average values
of the corresponding elements of the untransposed case.

APPENDIX E

URAM AND MILLER'S METHOD — THEORY

The Uram and Miller's method is a time domain method, in which by making certain assumptions, an incremental solution of the transmission line equations for the voltage and current at each instant at any point on a transmission line, subsequent to a switching operation is made possible.

The voltage and current vectors at any point on a transmission line, subsequent to a switching operation are governed by the equations

$$-\frac{\partial \underline{v}(x,t)}{\partial x} = [R]\,\underline{i}\,(x,t) + [L]\,\frac{\partial \underline{i}(x,t)}{\partial t}$$

$$-\frac{\partial \underline{i}(x,t)}{\partial x} = [C]\,\frac{\partial \underline{v}(x,t)}{\partial t}$$

neglecting the conductance to ground of the transmission line.

Taking the Laplace transform with respect to t, these equations become

$$-\frac{d\underline{V}(x,s)}{dx} = \{[R] + s[L]\}\underline{I}(x,s) \qquad (E.1)$$

$$-\frac{d\underline{I}(x,s)}{dx} = s[C]\,\underline{V}(x,s) \qquad (E.2)$$

These two equations can be combined to get

$$\frac{d^2\underline{V}(x,s)}{dx^2} = [Z][Y]\,\underline{V}(x,s) \qquad (E.3)$$

where $[Z] = \{ [R] + s[L] \}$

and $[Y] = s[C]$

The above coupled matrix differential equation can be decoupled by performing the modal transformation,

$$\underline{V}(x,s) = [T] \underline{F}(x,s)$$

where $[T]$ is an eigenvector matrix of the matrix product $[Z][Y]$.

Equation (E.3), after the modal transformation, takes the form

$$\frac{d^2\underline{F}(x,s)}{dx^2} = [T]^{-1} [Z][Y][T] \underline{F}(x,s)$$

where $[T]^{-1}[Z][Y][T]$ is a diagonal matrix.

The solution of this equation for the ith modal component can be written as

$$F_i(x,s) = K_{1i}(s) \, e^{-\sqrt{(Z_{mi} Y_{mi})}x} + K_{2i}(s) \, e^{\sqrt{(Z_{mi} Y_{mi})}x}, i=1,N.$$

where $K_{1i}(s)$ and $K_{2i}(s)$ are constants, independent of x and $Z_{mi} Y_{mi}$ = ith diagonal element of the diagonal matrix $[T]^{-1}[Z][Y][T]$.

In the vector notation, the solution can be written as

$$\underline{F}(x,s) = \underline{K_1}(s) \, \underline{e}_x^- + \underline{K_2}(s) \, \underline{e}_x^+$$

where

$$\underline{K}_1(s)\,e_x^- = [K_{11}(s)\,e^{-\sqrt{(Z_{m1}\,Y_{m1})}x} \quad \ldots \quad K_{1N}(s)\,e^{-\sqrt{(Z_{mN}\,Y_{mN})}x}]^T$$

and

$$\underline{K}_2(s)\,e_x^+ = [K_{21}(s)\,e^{\sqrt{(Z_{m1}\,Y_{m1})}x} \quad \ldots \quad K_{2N}(s)\,e^{\sqrt{(Z_{mN}\,Y_{mN})}x}]^T$$

Transforming back to the phase quantities

$$\underline{V}(x,s) = [T]\,\underline{F}(x,s) = [T]\,\{\underline{K}_1(s)e_x^- + \underline{K}_2(s)e_x^+\}$$

From equation (E.1), the solution for the transformed phase current vector can be written as

$$\underline{I}(x,s) = [T]\,[\Omega]^{-1}\{\underline{K}_1(s)e_x^- - \underline{K}_2(s)e_x^+\}$$

where $[\Omega] = \begin{bmatrix} \sqrt{(Z_{m1}/Y_{m1})} & 0 & \ldots & 0 \\ 0 & \sqrt{(Z_{m2}/Y_{m2})} & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \ldots & \sqrt{(Z_{mN}/Y_{mN})} \end{bmatrix}$

From these general transformed voltage and current vector solutions, the expressions for the sending end and receiving end transformed voltage and current vectors can be written as

$$\underline{V}_s(s) = [T]\,\{\underline{K}_1(s) + \underline{K}_2(s)\} \tag{E.4}$$

$$\underline{I}_s(s) = [T][\Omega]^{-1}\{\underline{K}_1(s) - \underline{K}_2(s)\} \tag{E.5}$$

$$\underline{V}_R(s) = [T]\,\{\underline{A}(s) + \underline{B}(s)\} \tag{E.6}$$

$$\underline{I}_R(s) = [T][\Omega]^{-1}\{\underline{A}(s) - \underline{B}(s)\} \tag{E.7}$$

where the ith components of the vectors $\underline{A}(s)$ and $\underline{K}_2(s)$ are related to the ith components of $\underline{K}_1(s)$ and $\underline{B}(s)$ by the expressions

$$A_i(s) = K_{1i}(s)\, e^{-\sqrt{(Z_{mi}\, Y_{mi})}\,\ell}$$

and

$$K_{2i}(s) = B_i(s)\, e^{-\sqrt{(Z_{mi}\, Y_{mi})}\,\ell}$$

In the above expressions

$$\ell = \text{the length of the line}$$

$$\sqrt{(Z_{mi}\, Y_{mi})} = \sqrt{[(R_{mi} + sL_{mi})sC_{mi}]}$$

$$= s\sqrt{(L_{mi}\, C_{mi})}\,(1 + \frac{R_{mi}}{sL_{mi}})^{\frac{1}{2}}$$

where R_{mi}, L_{mi} and C_{mi} are the resistance, inductance and capacitance per unit length respectively offered by the line for the ith mode of propagation.

If $\dfrac{R_{mi}}{sL_{mi}} \ll 1$, $\sqrt{(Z_{mi}\, Y_{mi})} = s\sqrt{(L_{mi}\, C_{mi})} + \dfrac{R_{mi}}{2}\sqrt{(\dfrac{C_{mi}}{L_{mi}})}$

$$\sqrt{(\frac{Y_{mi}}{Z_{mi}})} = \sqrt{(\frac{C_{mi}}{L_{mi}})}\,(1 + \frac{R_{mi}}{sL_{mi}})^{-\frac{1}{2}}$$

If $\dfrac{R_{mi}}{sL_{mi}} \ll 1$, $\sqrt{(\dfrac{Y_{mi}}{Z_{mi}})} = \sqrt{(\dfrac{C_{mi}}{L_{mi}})}$.

Hence $A_i(s) = K_{1i}(s)\, e^{-(R_{mi}/2)\sqrt{(C_{mi}/L_{mi})}\,\ell}\, e^{-s\sqrt{(L_{mi}\, C_{mi})}\,\ell}$

$$\tag{E.8}$$

$$K_{2i}(s) = B_i(s)\, e^{-(R_{mi}/2)\sqrt{(C_{mi}/L_{mi})}\,\ell}\, e^{-s\sqrt{(L_{mi}\, C_{mi})}\,\ell}$$

$$\tag{E.9}$$

When equations (E.4) - (E.7) are transformed back to time domain, the sending end and receiving end voltage and current vectors become

$$\underline{v}_s(t) = [T] \{\underline{k}_1(t) + \underline{k}_2(t)\} \tag{E.10}$$

$$\underline{i}_s(t) = [T][\Omega]^{-1} \{\underline{k}_1(t) - \underline{k}_2(t)\} \tag{E.11}$$

$$\underline{v}_R(t) = [T] \{\underline{a}(t) + \underline{b}(t)\} \tag{E.12}$$

$$\underline{i}_R(t) = [T][\Omega]^{-1} \{\underline{a}(t) - \underline{b}(t)\} \tag{E.13}$$

where $[\Omega] =$

$$\begin{vmatrix} \sqrt{(L_{m1}/C_{m1})} & 0 & \cdots\cdots & 0 \\ 0 & \sqrt{(L_{m2}/C_{m2})} & 0 \cdots & 0 \\ \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot & \\ 0 & 0 & \cdots & \sqrt{(L_{mN}/C_{mN})} \end{vmatrix}$$

By the shifting property of Laplace transforms, when functions A_i and K_{2i} are related by expressions as in equations (E.8) and (E.9) to the functions $K_{1i}(s)$ and $B_i(s)$ in the Laplace domain, the corresponding inverse transformed functions in time domain are related by the expressions

$$a_i(t) = e^{-(R_{mi}/2)\sqrt{(C_{mi}/L_{mi})}\ell} \; k_{1i}(t-\tau_i) \; u(t-\tau_i) \;, \quad i=1,N$$

$$k_{2i}(t) = e^{-(R_{mi}/2)\sqrt{(C_{mi}/L_{mi})}\ell} \; b_i(t-\tau_i) \; u(t-\tau_i) \;, \quad i=1,N$$

where τ_i = the travel time of the ith mode of propagation of the travelling wave to traverse the length of the line

$$= \sqrt{(L_{mi} \ C_{mi})} \ell \ .$$

Thus the voltage and current vectors at each instant of time at any point on a transmission line, subsequent to a switching operation can be expressed in terms of the values of the four function vectors $\underline{k}_1(t)$, $\underline{k}_2(t)$, $\underline{a}(t)$ and $\underline{b}(t)$ at that instant. Among these four function vectors, $\underline{a}(t)$ and $\underline{k}_2(t)$ have been shown to be the attenuated delay functions of the other two function vectors $\underline{k}_1(t)$ and $\underline{b}(t)$ respectively. The function vectors $\underline{k}_1(t)$ and $\underline{b}(t)$ can be expressed in terms of the function vectors $\underline{a}(t)$ and $\underline{k}_2(t)$ by the terminal conditions of the line. For the case of the line, which was relaxed before the switching operation, initially till the time equal to the travelling time of the line, the delayed functions are zero.

i.e. $a_i(t) = 0$, $i = 1, N$ for $0 < t < \tau_i$

and $k_{2i}(t) = 0$, $i = 1, N$ for $0 < t < \tau_i$.

Thus an incremental solution of the transmission line equations in time domain is possible for an initially relaxed line.

APPENDIX F

FLOW CHART FOR COMPUTATION OF SEQUENTIAL CLOSING TRANSIENTS BY MODIFIED FOURIER TRANSFORM METHOD

Read Data

$a, \Delta t, \Delta\omega, R_s, L_s, T_1, T_2, T_3, \Omega, \omega_s$

$[Q], [Q]^{-1}$, Line data

$\omega = 0$

Calculate and Store

$$E_{s1}(\omega) = (a+j\omega)/[(a+j\omega)^2 + \omega_s^2]$$

$$E_{s2}(\omega) = e^{-(a+j\omega)T_1}[(a+j\omega)\cos(\omega_s T_1 + \tfrac{2\pi}{3})$$

$$-\omega_s \sin(\omega_s T_1 + \tfrac{2\pi}{3})]/[(a+j\omega)^2 + \omega_s^2]$$

$$E_{s3}(\omega) = e^{-(a+j\omega)T_2}[(a+j\omega)\cos(\omega_s T_2 - \tfrac{2\pi}{3})$$

$$-\omega_s \sin(\omega_s T_2 - \tfrac{2\pi}{3})]/[(a+j\omega)^2 + \omega_s^2]$$

$$[A(\omega)] = [Z(\omega)]^{-1}[\psi(\omega)][\coth\psi\ell]$$

$$[B(\omega)] = -[Z(\omega)]^{-1}[\psi(\omega)][\operatorname{Cosech}\psi\ell]$$

$$[X(\omega)] = [A(\omega)] - [B(\omega)][A(\omega)]^{-1}[B(\omega)]$$

$$[Z(\omega)] = [B(\omega)]^{-1}[A(\omega)]$$

$$[Y(\omega)] = [B(\omega)] - [A(\omega)][Z(\omega)]$$

$\omega = \omega + \Delta\omega$

$\omega > \Omega$

NO

YES

10

10

IIII = 1

$\omega = 0$
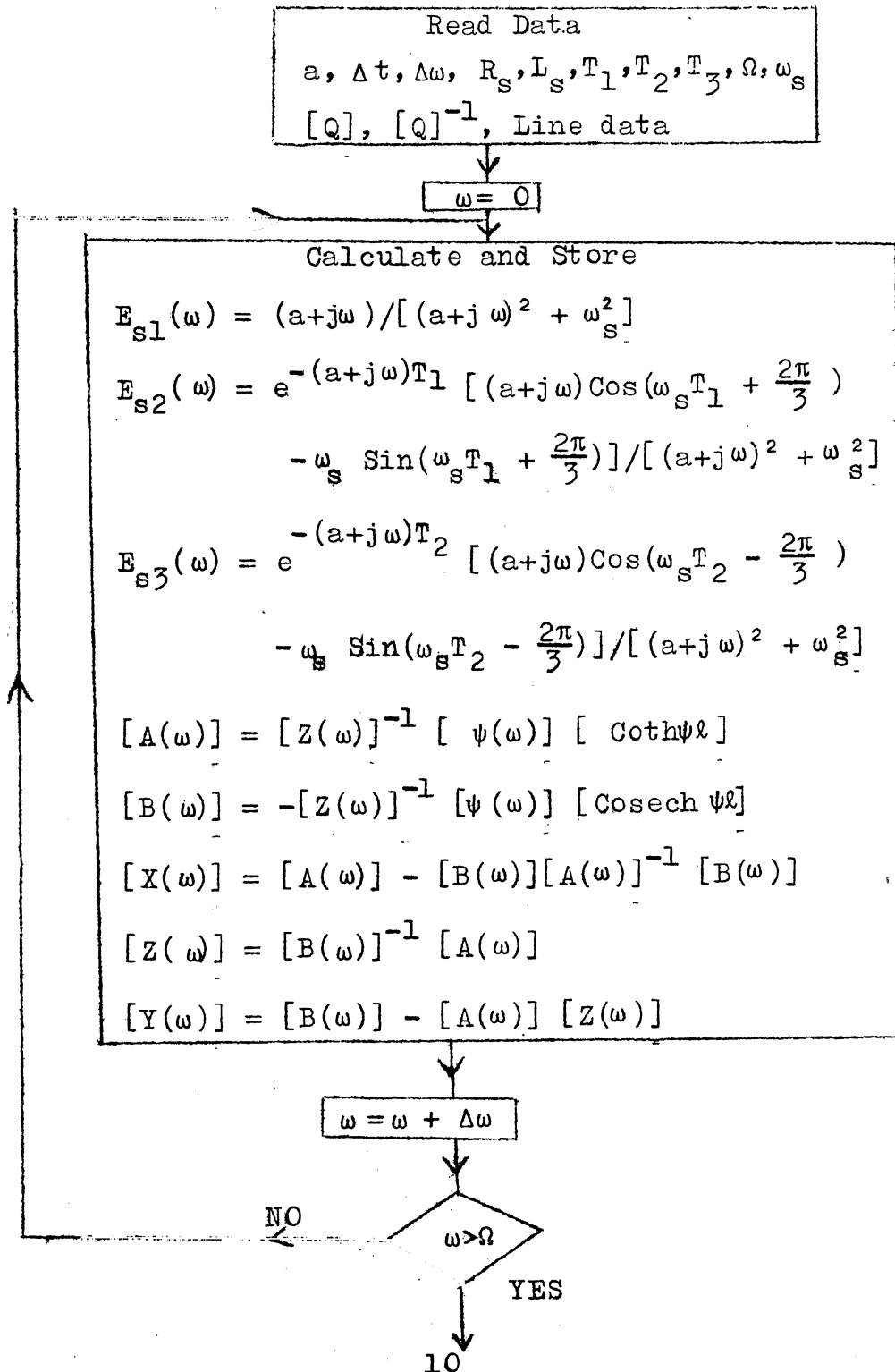
$T_{max} = T_1$

$\underline{v}_S(t) = 0, \ 0 < t < T_3$

$\underline{v}_R(t) = 0, \ 0 < t < T_3$

IIII = 1

NO

YES

1

$$[Y_s(\omega)] = \begin{array}{ccc} 1/[R_s+(a+j\omega)L_s] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$$

IIII=2

NO

YES

2

$\omega = 0$

$T_{max} = T_2$

3

$\omega = 0$

$T_{max} = T_3$

7

6

Calculate $V_{s2}(\omega)$, $V_{s3}(\omega)$, the modified Fourier transforms of $v_{s2}(t)$ in $0 < t < T_1$ and of $v_{s3}(t)$ in $0 < t < T_2$

Calculate $V_{s2}(\omega)$, the modified Fourier transform of $v_{s2}(t)$ in the interval $0 < t < T_1$

$$[Y_s(\omega)] = \begin{array}{ccc} 1/[R_s+(a+j\omega)L_s] & 0 & 0 \\ 0 & 1/[R_s+(a+j\omega)L_s] & 0 \\ 0 & 0 & 0 \end{array}$$

$$[Y_s(\omega)] = \begin{array}{ccc} 1/[R_s+(a+j\omega)L_s] & 0 & 0 \\ 0 & 1/[R_s+(a+j\omega)L_s) & 0 \\ 0 & 0 & 1/[R_s+(a+j\omega)L_s] \end{array}$$

4

4

$$[Z_S(\omega)] = [X(\omega)] + [Y_S(\omega)]$$

If IIII=2   IIII = 1 or 2 or 3   If IIII=3

If IIII=1

$$\underline{I}_S(\omega) = \begin{array}{c} E_{s1}(\omega)/[R_S+(a+j\omega)\overline{L_S}] \\ 0 \\ 0 \end{array}$$

$$\underline{I}_S(\omega) = \begin{array}{c} [E_{s1}(\omega)/[R_S+(a+j\omega)L_S] \\ [E_{s2}(\omega)+V_{s2}(\omega)]/[R_S+(a+j\omega)L_S] \\ 0 \end{array}$$

NO   IIII=3   YES

$$\underline{I}_S(\omega) = \begin{array}{c} E_{s1}(\omega)/[R_S+(a+j\omega)L_S] \\ [E_{s2}(\omega)+V_{s2}(\omega)]/[R_S+(a+j\omega)L_S] \\ [E_{s3}(\omega)+V_{s3}(\omega)]/[R_S+(a+j\omega)L_S] \end{array}$$

Calculate

$$[Z_R(\omega)] = [Y(\omega)]-[Y_S(\omega)] \ [Z(\omega)]$$
$$\underline{P}_R(\omega) = \text{Real Part} \ \{ [Z_R(\omega)]^{-1} \ \underline{I}_S(\omega)\}$$

Calculate

$$\underline{P}_S(\omega) = \text{Real Part} \ \{ [Z_S(\omega)]^{-1} \ \underline{I}_S(\omega)\}$$

$$t = 0$$

8

$$t = t + \Delta t$$

5

5

NO ← ◇ IIII = 3 ◇

YES

$$\underline{V}_R(t,\omega) = \frac{2e^{at}}{\pi}\, \underline{P}_R(\omega)\, \text{Cos}\,\omega t\; \frac{\text{Sin}(\frac{\pi\omega}{\Omega})}{(\frac{\pi\omega}{\Omega})}$$

$$\underline{V}_S(t,\omega) = \frac{2e^{at}}{\pi}\, \underline{P}_S(\omega)\, \text{Cos}\,\omega t\; \frac{\text{Sin}(\frac{\pi\omega}{\Omega})}{(\frac{\pi\omega}{\Omega})}$$

YES ← ◇ $\omega = 0$ ◇

NO

NO ← ◇ IIII = 3 ◇

YES

Calculate and store

$$\underline{v}_R(t) = \underline{v}_R(t) + \frac{\Delta\omega}{2}[\underline{V}_R(t,\omega) + \underline{V}_R(t,\omega-\Delta\omega)]$$

Calculate and store

$$\underline{v}_S(t) = \underline{v}_S(t) + \frac{\Delta\omega}{2}\,[\underline{V}_S(t,\omega) + \underline{V}_S(t,\omega-\Delta\omega)]$$

◇ $t > T_{max}$ ◇ → NO → Go To 8

YES

YES ← ◇ $\omega = \Omega$ ◇

Go To 20

NO

$\omega = \omega + \Delta\omega$

If IIII=2 ← ◇ IIII = 1 or 2 or 3 ◇ → If IIII=1

Go to 6 → Go To 1

If IIII=3

20

YES

IIII ≤ 2

NO

Print $\underline{v}_s(t)$, $\underline{v}_R(t)$

for $0 < t < T_3$

IIII = IIII+1

If IIII=3
Go To 3

IIII = 2 or 3 or 4

If IIII=2

Go To 2

If IIII=4

Stop
End

APPENDIX   G

OBRA-LUCKNOW LINE DATA

Length of the line   400 Kms

Shunt reactors of reactance value 3200 ohms at both the ends.

Source reactance at Obra end is 67.8 ohms, which takes in-
to account both the local generation, short circuit power
supplied through transmission lines on   lower voltage levels
and the short circuit power of the 400 KV transformers.

The line parameters are as follows:

Positive sequence resistance   0.027 ohm/Km

Positive sequence inductive reactance 0.331 ohm/Km

Positive sequence capacitance 10.95×10^{-9} f/Km

Zero sequence resistance   0.261 ohm/Km

Zero sequence inductive reactance   1.31 ohm/Km

Zero sequence capacitance 8.42×10^{-9} f/Km.

CURRICULUM VITAE

1. Candidate's name: R. Balasubramanian

2. Academic Background:

Degree	Specialisation	Institute	Year
B.E.	Electrical Engineering	P.S.G. College of Technology, Madras University	1968
M.Sc.(Engg.)	Power Systems	Annamalai University	1970

3. Publications:

i)   Calculation of transient due to fault initiation on a double circuit transmission line, Accepted for publication in the Proceedings IEE, London.

ii) Calculation of energising transient on a double circuit transmission line, Submitted for publication in Proceedings IEE-IERE (India).

iii) Accurate evaluation of energising transients on double circuit EHV transmission lines, Submitted for presentation in IEEE Winter Power Meeting, 1977.